

Lecture on

Compressor

by

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What is Compressor ?

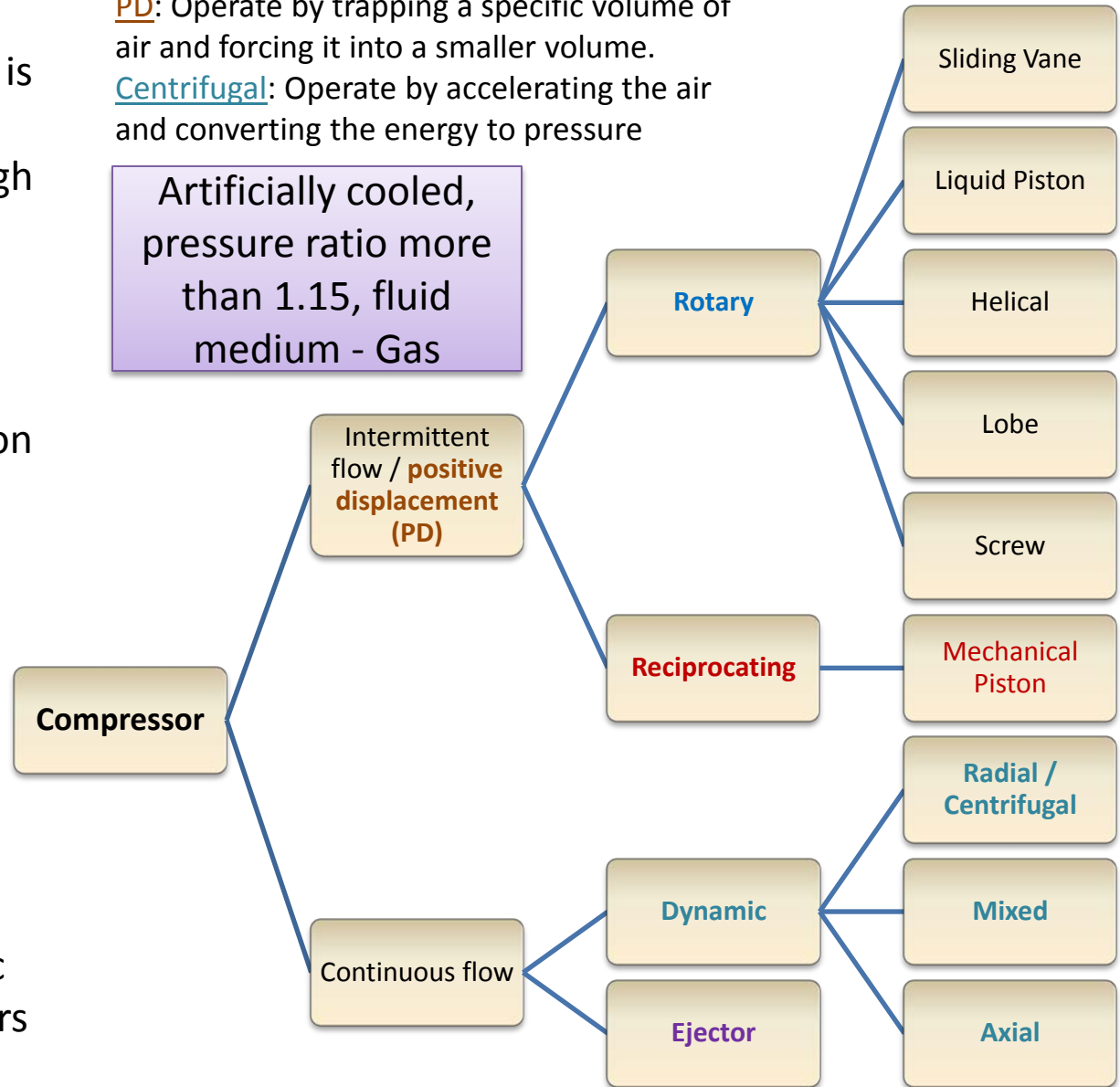
Compressor is a device which is used to increase the pressure of air from low pressure to high pressure by using some external energy

- For filling the air in tube of vehicles
- In automobile service station to clean vehicles.
- For spray painting in paint industries.
- In vehicle to operate air brakes.
- For cleaning workshop machines.
- For supercharging of an IC engines.
- For operation of pneumatic tools i.e. rock drills, vibrators etc.

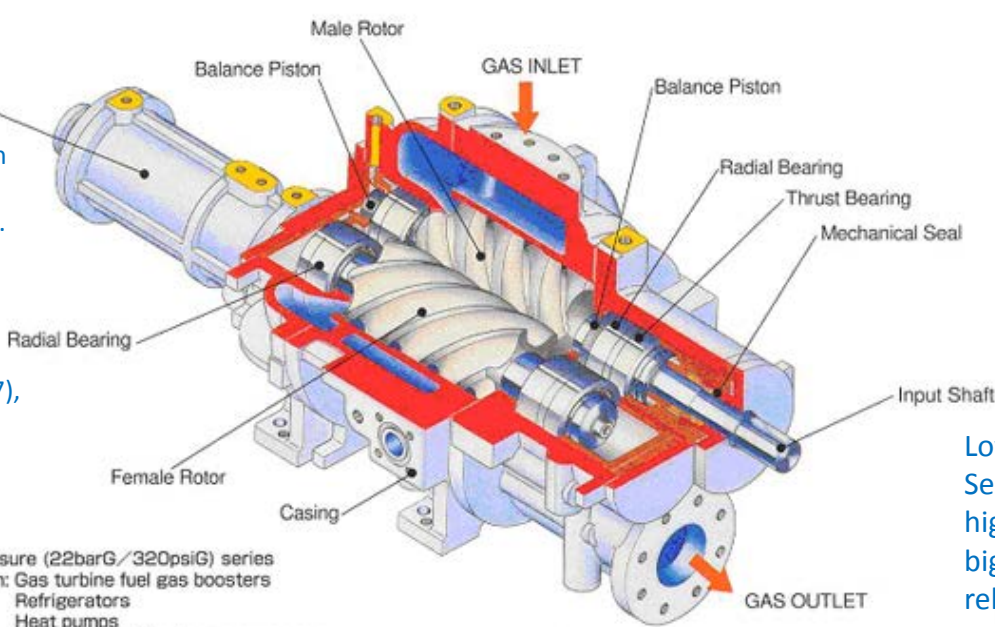
PD: Operate by trapping a specific volume of air and forcing it into a smaller volume.

Centrifugal: Operate by accelerating the air and converting the energy to pressure

Artificially cooled,
pressure ratio more
than 1.15, fluid
medium - Gas

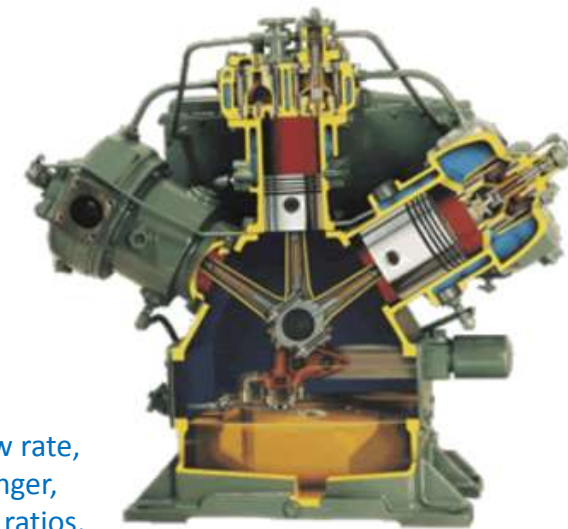


- Quiet operation
- High volume of air, steady flow.
- Lower energy cost, small size
- Suitable for continuous operation (24/7),
- low efficiency
- Low pressure ratio



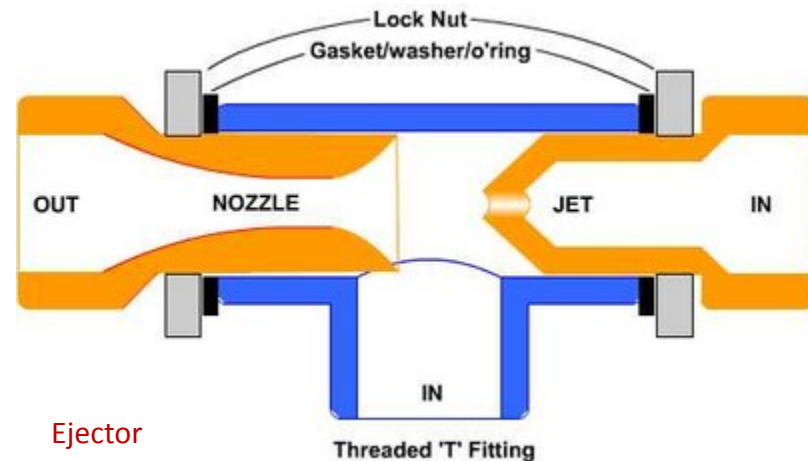
Middle-high pressure (22barG/320psiG) series
 Major application: Gas turbine fuel gas boosters
 Refrigerators
 Heat pumps
 Oil and gas gathering compressors
 LNG boil off gas compressors

Rotary – screw compressor



Low mass flow rate,
 Service life longer,
 high pressure ratios,
 bigger size, and is
 relatively cheap.

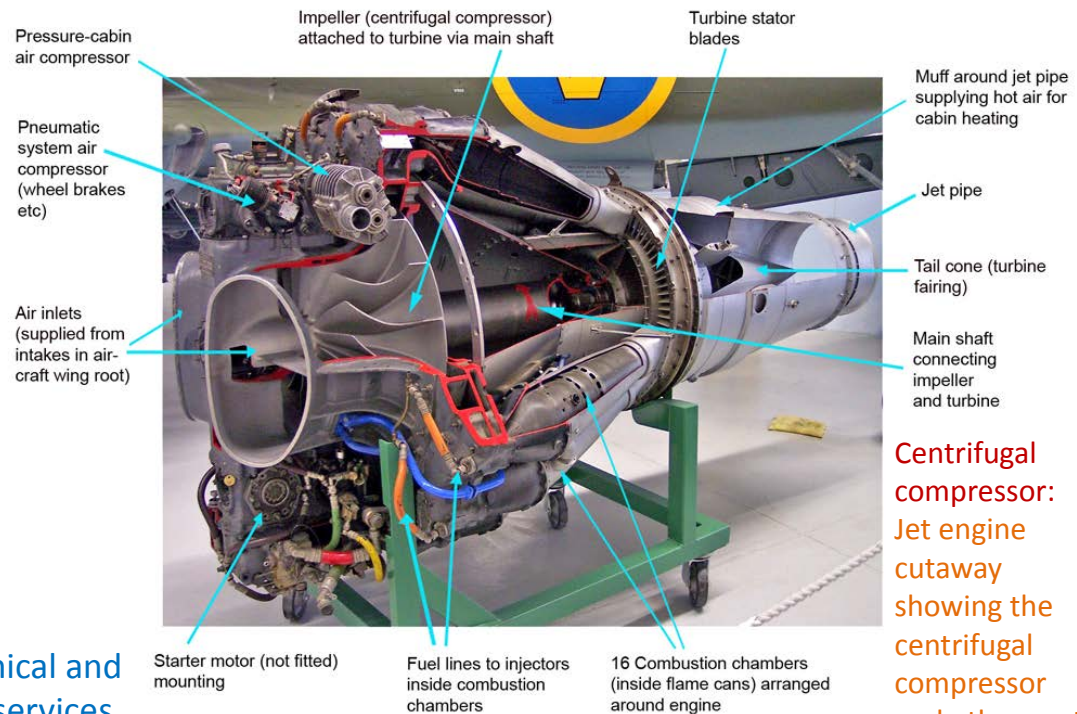
Reciprocating



Ejector

Process plant optimization. Gas compression.
 Production boosting.

Centrifugal compressor is widely used in chemical and petroleum refinery industry for specific services.



Centrifugal compressor:
 Jet engine cutaway showing the centrifugal compressor and other parts

Definitions of Compressor

Compression ratio:- It is defined as the ratio of volume of air before compression to the volume of air after compression.

Compressor capacity:- It is the quantity of air actually delivered by a compressor in m^3/minute or m^3/sec .

Free air Delivered(FAD):- It is the volume of air delivered by compressor under the compressor intake conditions (i.e. temperature and pressure).

Swept Volume:- The volume displaced or swept by piston when it moves between top dead center and bottom dead center.

Clearance volume:- it is the difference between the total volume and the swept volume, basically the gap that remains between the piston head and the cylinder head when at top dead center.

Efficiencies:

Volumetric efficiency:-

It is the ratio of actual volume of the FAD at standard atmospheric condition in one delivery stroke (Actual air intake) to the swept volume (theoretical air intake) by the piston during the stroke.

Isothermal efficiency:-

It is defined as the ratio of isothermal power (P_{iso}) (i.e. required input power at isothermal process) done to the indicated power (IP) or actual work done.

Mechanical efficiency:-

It is the ratio of indicated power (IP) to the shaft(Brake) Power (P_{shaft}).

Overall efficiency:-

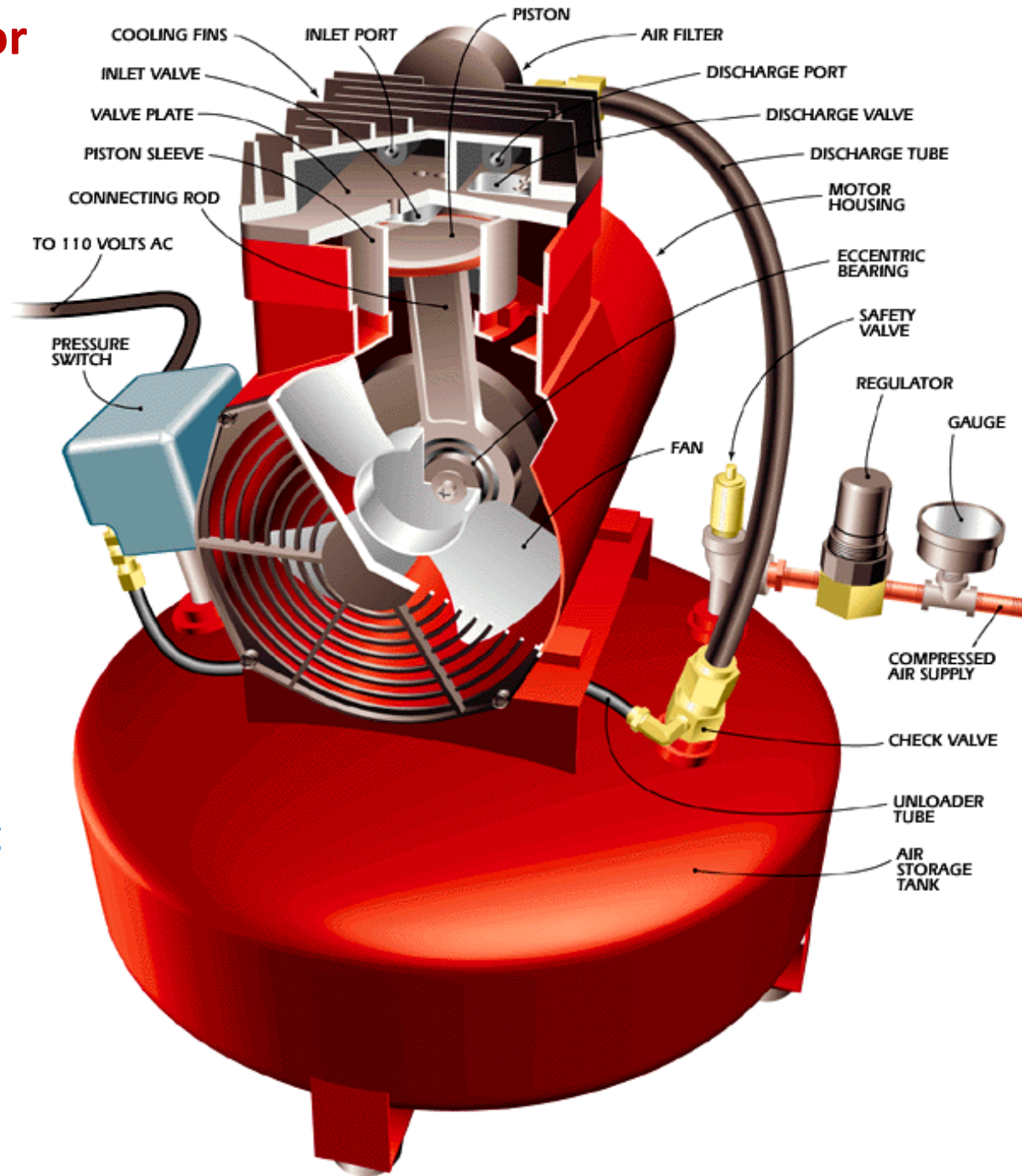
It is the ratio of isothermal power (P_{iso}) to the shaft(Brake) Power (P_{shaft}).

The Analysis Objectives

- To calculate air pressure and temperature before and after compression process
- To calculate indicated work/power
- To obtain compressor efficiency
- To calculate free air delivery rate (kg/s or m³/s)

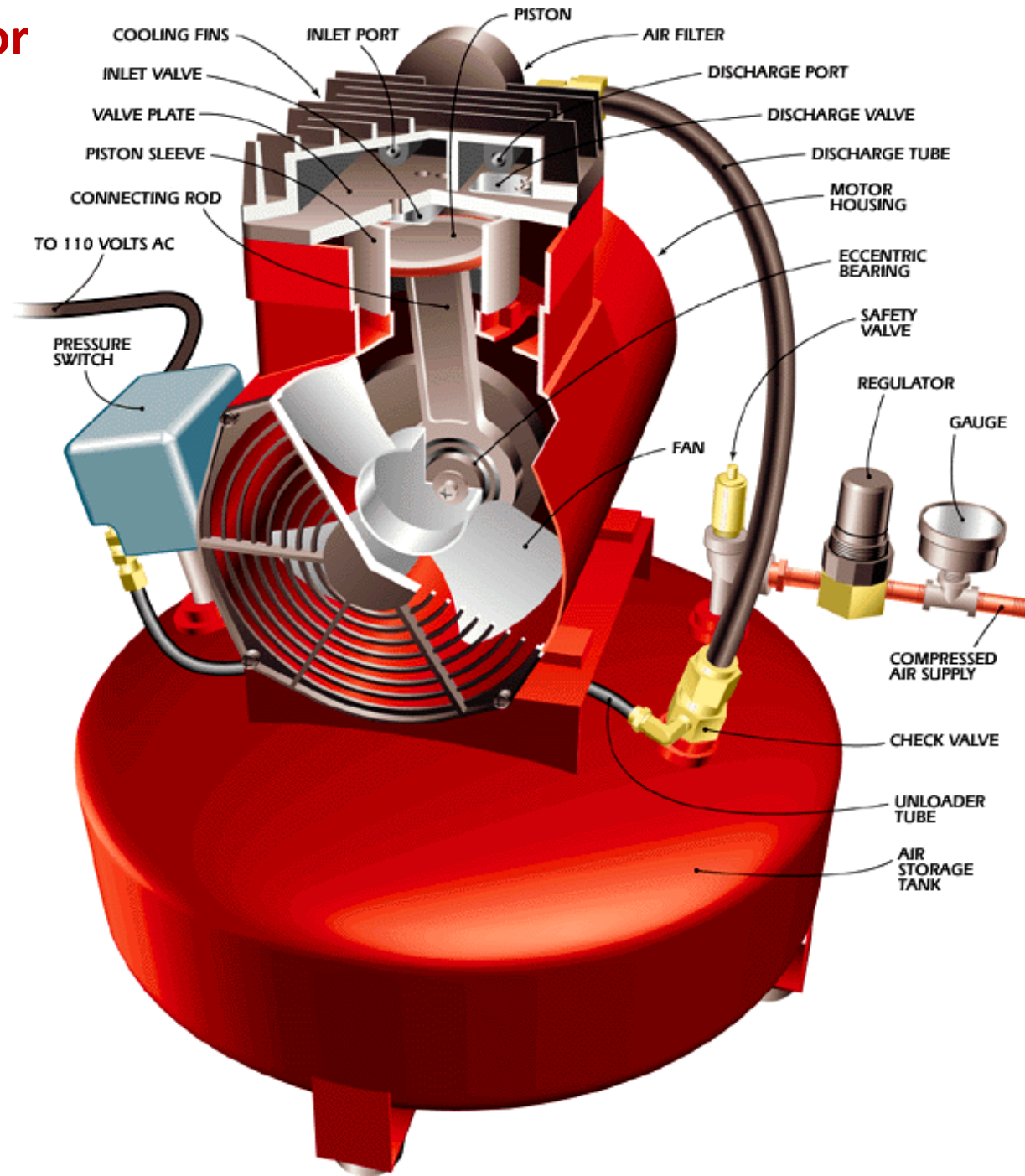
Reciprocating Compressor

The primary components of a typical reciprocating compressor system can be seen in Figures. The compression cylinders, also known as stages, of which a particular design may have from one to six or more, provide confinement for the process gas during compression. A piston is driven in a reciprocating action to compress the gas. Arrangements may be of single- or dual-acting design. (In the dual-acting design, compression occurs on both sides of the piston during both the advancing and retreating stroke.) Some dual-acting cylinders in high-pressure applications will have a piston rod on both sides of the piston to provide equal surface area and balance loads.



Reciprocating Compressor

Tandem cylinder arrangements help minimize dynamic loads by locating cylinders in pairs, connected to a common crankshaft, so that the movements of the pistons oppose each other. Gas pressure is sealed and wear of expensive components is minimized through the use of disposable piston rings and rider bands respectively. These are formed from comparatively soft metals relative to piston and cylinder/liner metallurgy or materials such as polytetrafluoroethylene (PTFE).



Reciprocating Compressor

In a reciprocating compressor, a volume of air is drawn into a cylinder, it is trapped, and compressed by piston and then discharged into the discharge line. The cylinder valves control the flow of air through the cylinder; these valves act as check valves.

Single – Acting compressor

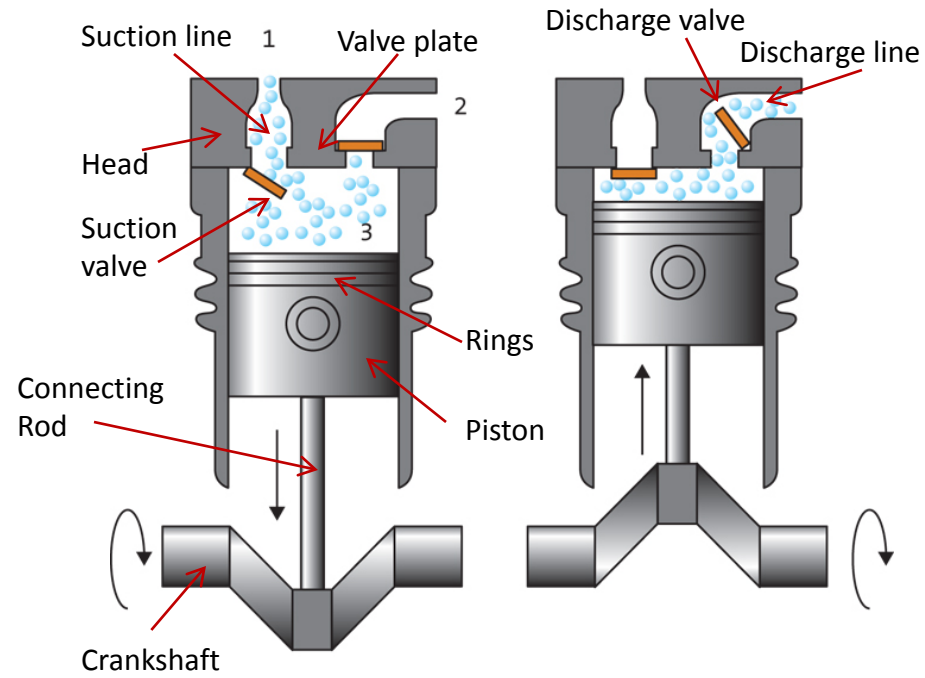
It is a compressor that has one discharge per revolution of crankshaft.

Double – Acting Compressor

It is a compressor that completes two discharge strokes per revolutions of crankshaft. Most heavy-duty compressors are double acting.

Multi-staging :Reduction in power required to drive the compressor.

- Better mechanical balance of the whole unit and uniform torque.
- Increase in volumetric efficiency.
- Reduced leakage loss.
- Less difficulty in lubrication due to low working temperature.
- Lighter cylinders can be used.
- Cheaper materials can be used for construction as the operating temperature is lower.

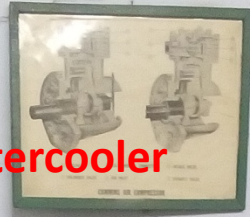


Front Face View



Manometer Reading (orifice)

TWO STAGE RECIPROCATING AIR COMPRESSOR
1. Three phase A.C. Induction Motor 25 H.P.
2. H.P. Cylinder Diameter 100 mm.
3. L.P. Cylinder Diameter 150 mm.
4. Stroke Length 100 mm.
5. Crank Diameter 100 mm.
6. Crank angle 180°
7. Transmission



1st Stage cylinder

Intercooler

2nd Stage cylinder guard

Dial weight Reading (Kg)

Mechanical dynamometer

T1, P1

Silencer cum filter

Air Box Unloader valve

Crank Case: Splash type Lubricating Oil indicator

Receiver Tank

Water outlet Drain Cock

Pressure control Switch

P3

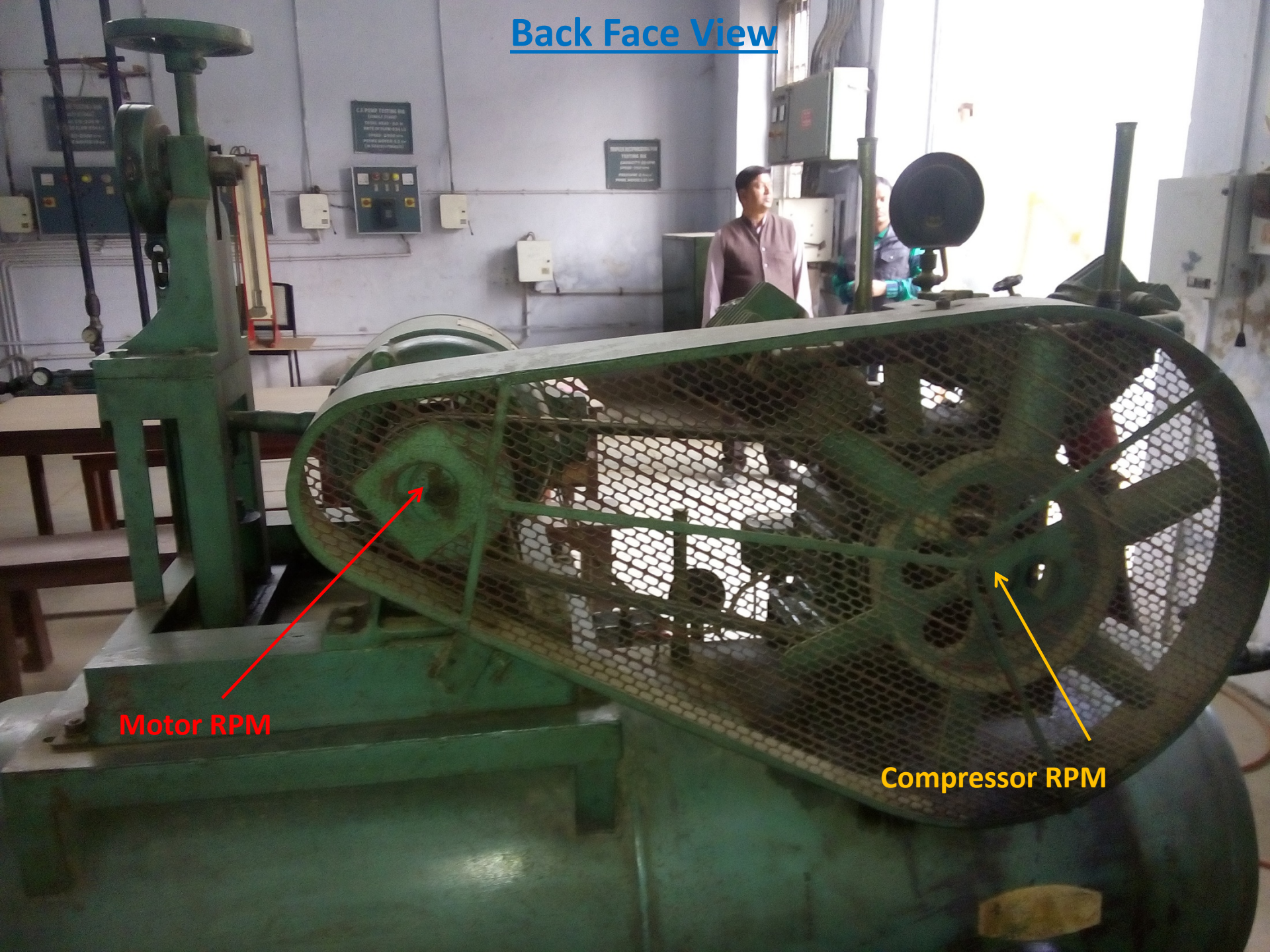
Safety Valve

Main Switch

Thermo Motor (without grouting i.e. Stand/support)

Motor (without grouting) to produce opposite torque for mechanical dynamometer.
Unloader valve: to keep the starting compressor operation in ambient pressure.

Back Face View



Motor RPM

Compressor RPM

OBSERVATIONS:

$$BHP = \frac{W \times N}{2000}$$

S/N	MANOMETER READING (Cm)			T <i>T_{atm}</i> (KG)	MOTOR RPM (N _m)	COMP. RPM (N _c)	P (Kg/Cm ²)	TEMPERATURE OF INTERCOOLER	
	LII	RII	DIFF					Inlet (T ₂) °C	Outlet (T ₃) °C
	(H ₁)	(H ₂)	(H=H ₁ -H ₂)						
1.									
2.									
3.									
4.									
5.									
6.									

CALCULATIONS:

1) VOLUMETRIC EFFICIENCY:

1. ACTUAL AIR INTAKE

Equivalent air column, $(H_e) = \frac{H \times W_w}{W_a}$ H = height of water column in metre

$W_w =$ specific weight of water (1000 Kg/M³), $W_a =$ specific weight of air (1.23 Kg/M³)

Diameter of orifice (d) = 0.02 m,

Area of orifice (A) = $\frac{\pi d^2}{4} = \frac{\pi (0.02)^2}{4}$

Volume of actual air intake (V_a) = $C_d \cdot A \cdot \sqrt{2gH_e}$, Where $C_d = 0.62$ \Rightarrow using column (3)

2. THEORETICAL AIR INTAKE

Piston diameter (D) = $\frac{89 \text{ mm}}{100 \text{ mm}}$, Stroke length (L) = $\frac{89 \text{ mm}}{100 \text{ mm}}$ Speed (N) = R.P.M

Volume of theoretical air intake (V_t) = $\frac{\pi \times D^2 \times L \times N}{4 \times 60}$ \Rightarrow using column (6)

Volumetric efficiency = $\frac{\text{Actual volume } (V_a)}{\text{Theoretical volume } (V_t)} \times 100\%$ $\approx 40\%$

III] ISOTHERMAL EFFICIENCY:

Isenthalpic H.P = $\frac{10^4 \times 1.03 \times V_a \times L \times \log(P_3/P_1)}{75}$

Where

P_3 = Delivery pressure in absolute unit = (P) = from column (7)

P_1 = Inlet pressure in absolute unit = $\text{atm} = 1 \text{ bar} = ?$

Input shaft power = $K \times \frac{N_m}{60} \times T \times \text{Transmission efficiency } \times (g)$

Where K = Dynamometer constant = 0.0005 & transmission efficiency = 0.9 (Say)

Isenthalpic efficiency = $\frac{\text{Isenthalpic H.P.}}{\text{Input shaft power}} \times 100$

III] HEAT REJECTED BY INTERCOOLER:

Heat rejected by intercooler = $m \times C_p \times (T_2 - T_3)$

Where $m = \frac{1.03 \times 10^4 \times V_a}{29.3 \times (t_a + 273)}$, $C_p = 0.24 \text{ Kcal/Kg}^\circ\text{K}$ & $t_a = \text{Room temperature}$

Reciprocating Compressor

Assumptions

The **working fluid** is assumed as a **perfect gas** and P - v - T can be calculated by using **simple equation of state**. Usually, these assumptions are used to calculate estimate pressure, P , volume, V , and temperature, T , of the working fluid.

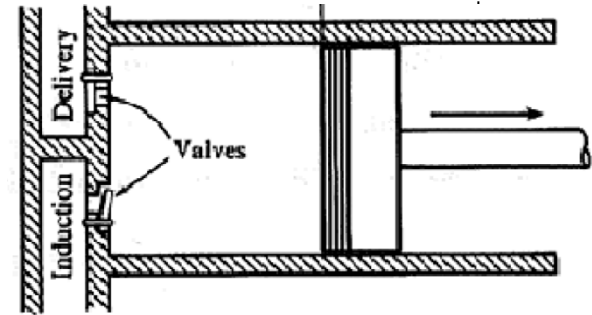
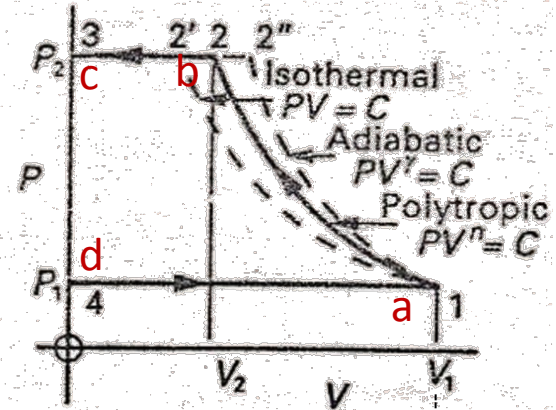
$$PV = mRT \quad PV/T = K_1 \quad PV^n = K_2$$

- Compressor without clearance volume
- Compressor with clearance volume
- Multistage compressors

Compressor without clearance volume

The Cycle of Operation

- The cycle of operation of a reciprocating air-compressor is best shown on a pressure-volume (p - V) *diagram*.
- It is known as an *indicator diagram* for the compressor.
- The cycle comprises of three processes:
 - $d - a$: An *induction stroke*
 - $a - b$: A *compression stroke*
 - $b - c$: A *delivery stroke*



Reciprocating Compressor

Compressor without clearance volume

Description of the Processes

d – a: The induction stroke

Intake valve opens, while exhaust valve closed.

Atmospheric air is drawn into the cylinder at constant pressure p_1 and temperature T_1 . Ideally, there is no heat loss to the surrounding from the air.

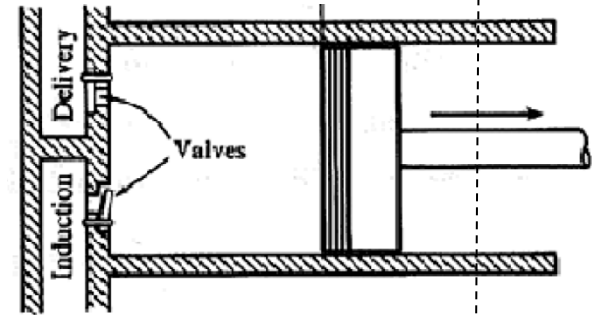
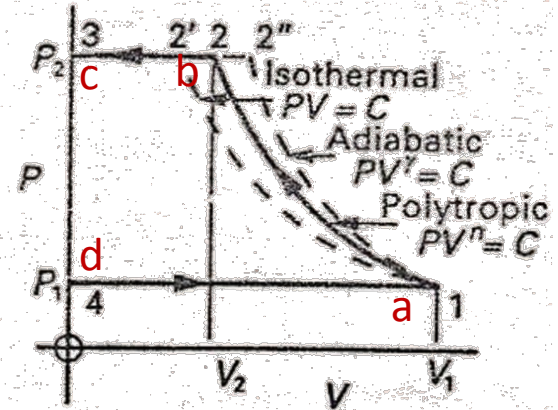
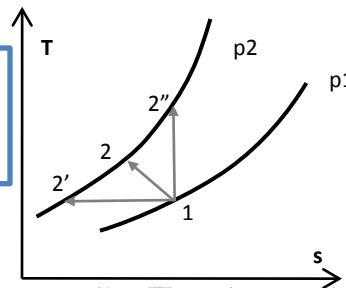
a – b: The compression stroke

Both intake and exhaust valves closed. The air is compressed according to a polytropic law $pV^n = \text{constant}$. Its pressure is increased from p_1 to p_2 . The temperature is also increased from T_1 to T_2 .

b – c: The delivery stroke

The intake valve closed while the exhaust valve opens. The compressed air is pushed out of the cylinder at constant pressure p_2 and temperature T_2 . There is no heat loss from the air to the surroundings.

During compression, due to its excess temperature above the compressor surrounding, the air will lose some heat. Thus neglecting the internal effect of friction the index is less than γ (i.e. <1.4), the adiabatic index. If $n = \gamma = 1.4$, area under curve is biggest, i.e. reversible adiabatic or entropy constant. Since work must be put into an air compressor to run it, every effort is made to reduce this amount of work input. It is observed that if compression is along isothermal, work done is less (though in practical it is not possible). Isothermal is attempted by cooling the compressor either by adding cooling fan or water jacket.



Reciprocating Compressor

Compressor without clearance volume

Analysis of Cycle

Indicated work per cycle

The area under the p - V diagram represents the net or indicated work done on the air per cycle.

Indicated work / cycle = area a - b - c - d = area 1 - 2 - 3 - 4 - 1
 = area under 1 - 2 + area under 2 - 3 - area under 4 - 1

$$= \left(\frac{p_2 V_2 - p_1 V_1}{n-1} \right) + p_2 V_2 - p_1 V_1$$

$$W = \left(\frac{n}{n-1} \right) (p_2 V_2 - p_1 V_1)$$

This work must be done on compressor

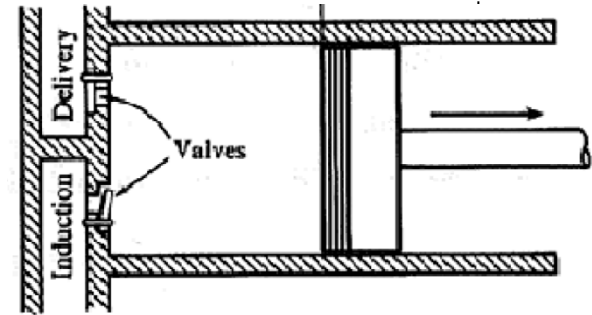
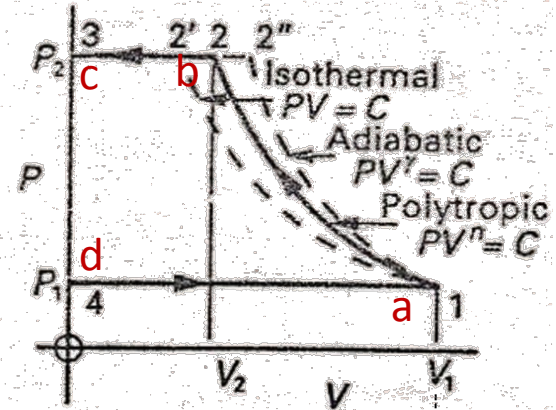
Assuming the air as a perfect gas,

$$p_1 V_1 = mRT_1 \quad p_2 V_2 = mRT_2$$

where m is the mass of air induced and delivered per cycle, R is the universal gas constant, where $R = 0.287$ kJ/kgK.

Substituting,

Indicated work / cycle $W = \left(\frac{n}{n-1} \right) mR(T_2 - T_1)$



$$\begin{aligned} W_{1-2} &= \int_1^2 p dV, \quad pV^n = C \\ &= C \int_1^2 V^{-n} dV = \frac{1}{1-n} (CV_2^{1-n} - CV_1^{1-n}) \\ &= \frac{1}{1-n} (p_2 V_2^n \times V_2^{1-n} - p_1 V_1^n \times V_1^{1-n}) \end{aligned}$$

Reciprocating Compressor

Compressor without clearance volume

Analysis of Cycle

Other form of the equation for indicated work/ cycle is

$$W = \left(\frac{n}{n-1} \right) mRT_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \left(\frac{n}{n-1} \right) p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Indicated Power (IP):

The indicated power (IP) is the **work done on the air per unit time**. The **mass flow per unit time \dot{m}** is often used to compute the work done/time or indicated power.

$$IP = \left(\frac{n}{n-1} \right) \dot{m} R (T_2 - T_1)$$

Mechanical efficiency:-

It is the ratio of indicated power to the shaft Power.

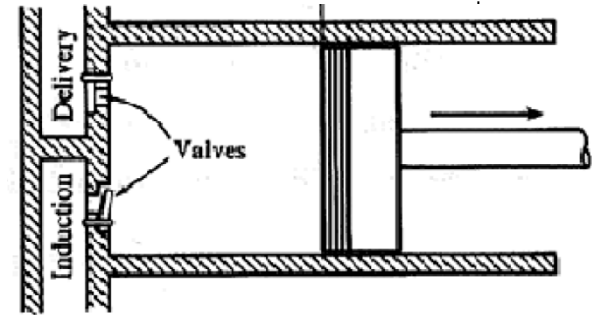
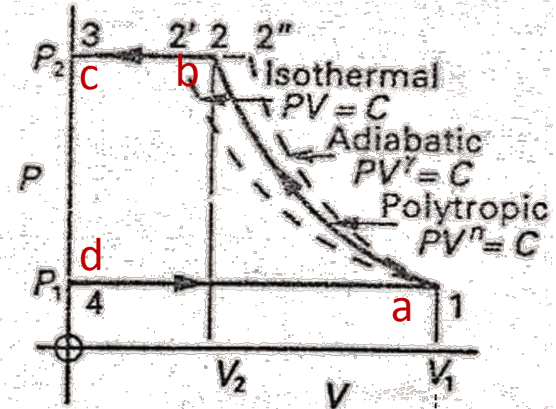
Shaft power = indicated power + friction power

Shaft power is the power supplied by the electric motor to compressor.

Motor efficiency:-

It is the ratio of shaft power to the input Power.

Input power is the electric power supplied to the electric motor



For Polytropic process

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad p_1 V_1^n = p_2 V_2^n$$

$$\frac{V_1}{V_2} = \left(\frac{p_2}{p_1} \right)^{\left(\frac{1}{n} \right)} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\left(\frac{n-1}{n} \right)} \quad \frac{T_2}{T_1} = \left(\frac{V_2}{V_1} \right)^{(n-1)}$$

Reciprocating Compressor

Compressor without clearance volume

Isothermal efficiency:-

It is the ratio of isothermal power (P_{iso}) to indicated power (IP)

Isothermal Power:

$$P_{iso} = \dot{m}RT_1 \ln(p_2 / p_1)$$

$$n = 1 \quad PV = C \quad \text{process}$$

$$W = \int_1^2 \frac{C}{V} dV = C \ln \frac{V_2}{V_1}$$

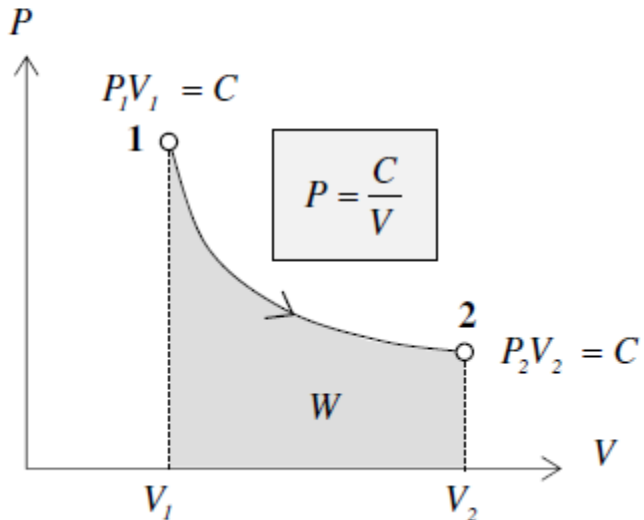
$$W = P_1 V_1 \ln \frac{V_2}{V_1} = P_2 V_2 \ln \frac{V_2}{V_1}$$

$$= P_2 V_2 \ln \frac{P_1}{P_2}$$

For ideal gas: $PV = mRT = C \Rightarrow T = \text{const}$

$$W = mRT \ln \frac{V_2}{V_1}$$

$$= mRT \ln \frac{P_1}{P_2}$$



Reciprocating Compressor

Compressor with clearance volume

Effect of the clearance volume is to reduce the volume actually aspirated.

Analysis of Cycle

It is a spacing between the top of the piston and the valve's heads when the piston is at the end of the delivery stroke. Good quality machines has a clearance volume of about 6%. But compressors with clearance of 30–35% are also common.

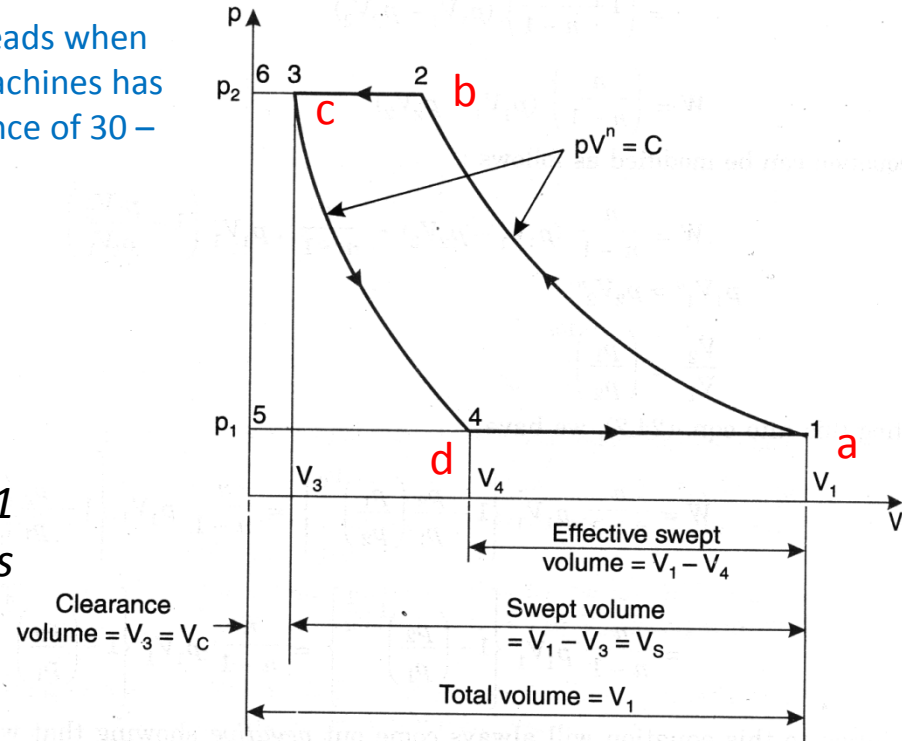
Processes

d – a: Induction process

The inlet valve opens. Fresh atmospheric air is induced into the cylinder at constant pressure p_1 and temperature T_1 . The volume of air induced is $(V_a - V_d)$. Ideally, there is no heat transfer from the air to the surroundings.

a – b: Compression process

Both valves closed. The induced air is compressed according to the polytropic law of $pV^n = \text{const.}$, until the pressure and temperature increases to p_2 and T_2 , respectively. Ideally, there is no heat transfer from the air to the surroundings of cylinder.



Clearance volume:

- Give a mechanical freedom to the moving parts
- Reduce noise and vibration during operation
- Prevent damage to moving components

Reciprocating Compressor

Compressor with clearance volume

V_c , residual gas

Deliver valve open, compress gas starts delivering from the cylinder

Analysis of Cycle

It is a spacing between the top of the piston and the valve's heads when the piston is at the end of the delivery stroke. Good quality machines has a clearance volume of about 6%. But compressors with clearance of 30 – 35% are also common.

Because of presence of clearance volume, volumetric efficiency is always less than unity, between 60% to 85%.

Processes

b – c: Delivery process

The exhaust valve opens. The compressed air is delivered out of the cylinder at constant pressure p_2 and temperature T_2 . Ideally, there is no heat transfer from the air to the surroundings.

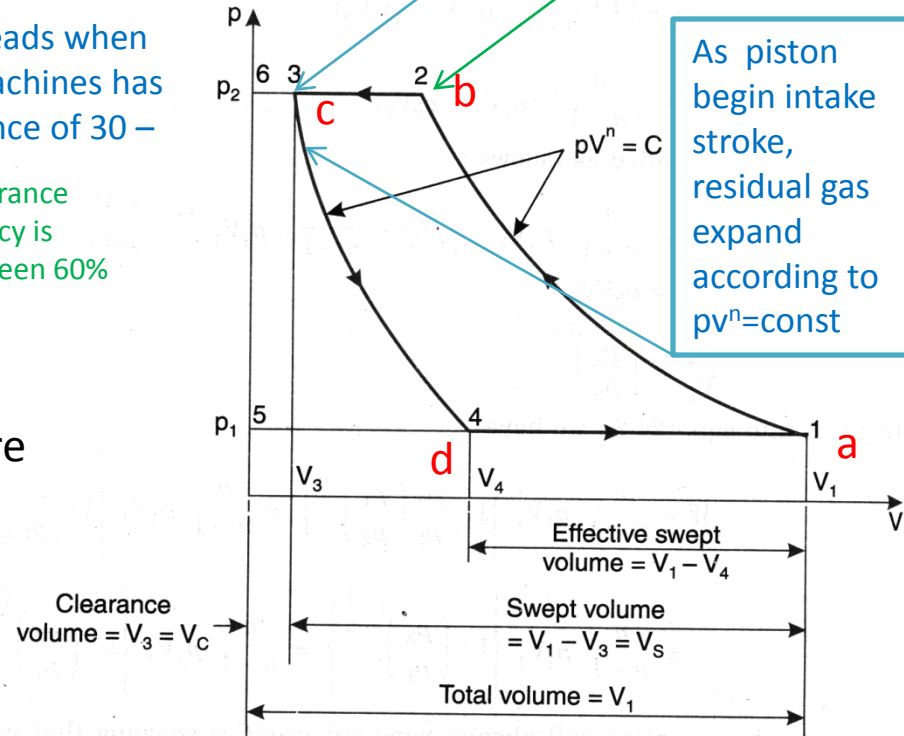
c – d: Expansion process

The piston begins the induction stroke. The compressed air occupying the clearance volume V_c expands according to the polytropic law of $pV^n = \text{const.}$, until the pressure and temperature fall to p_1 and T_1 , respectively. Ideally, there is no heat transfer from the air to the surroundings.

Note: At the end of the delivery stroke, the clearance volume V_c is filled with compressed air at pressure p_2 and temperature T_2 .

$$\text{Volumetric efficiency} = \frac{V_1 - V_4}{V_1 - V_3}$$

It is the ratio between FAD at standard atmospheric condition in one delivery stroke (Actual air intake) to the swept volume (theoretical air intake)



As piston begin intake stroke, residual gas expand according to $pV^n = \text{const}$

Reciprocating Compressor

Compressor with clearance volume

Assuming polytropic index to be same for both compression and clearance expansion **Indicated work / cycle =**

$$W = \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] - \left(\frac{n}{n-1}\right) p_4 V_4 \left[\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1 \right]$$

But $p_4 = p_1$, $p_3 = p_2$ therefore

$$W = \left(\frac{n}{n-1}\right) p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \left(\frac{n}{n-1}\right) (\dot{m}_a - \dot{m}_d) R [T_2 - T_1]$$

Volumetric efficiency =

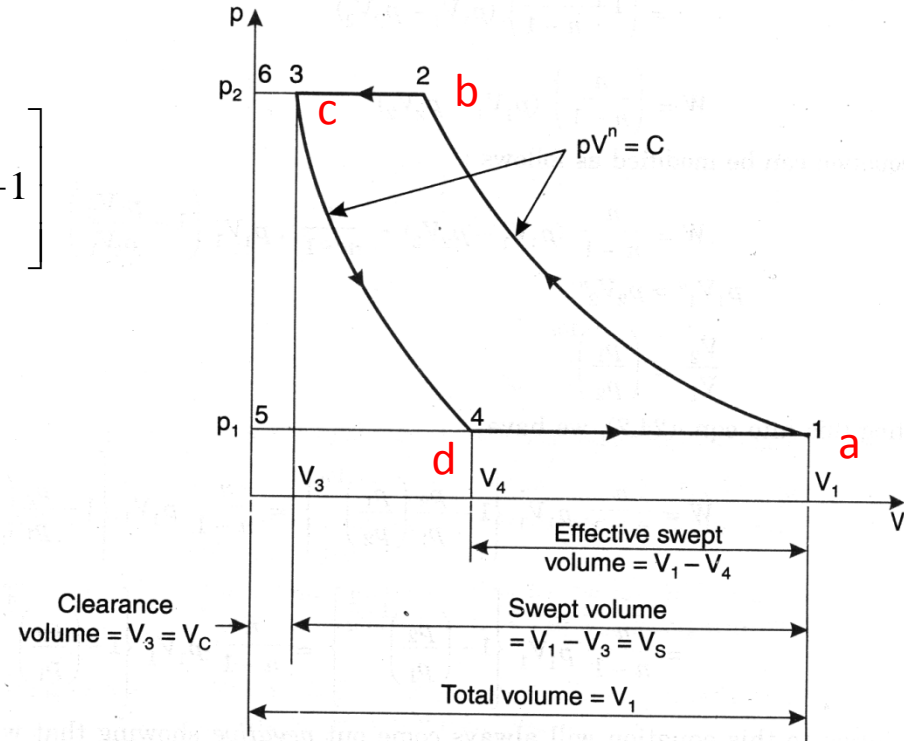
$$= \frac{V_1 - V_4}{V_1 - V_3} = \frac{V_1 - V_4}{V_s}$$

$$= 1 + k - k(V_1/V_2)$$

$$= 1 + k - k(p_2/p_1)^{1/n}$$

$$= 1 + k - k(p_3/p_4)^{1/n}$$

Where, $k = \text{clearance ratio} = V_3/(V_1 - V_3) = V_c/V_s$
Ratio has a value 4% - 10 %



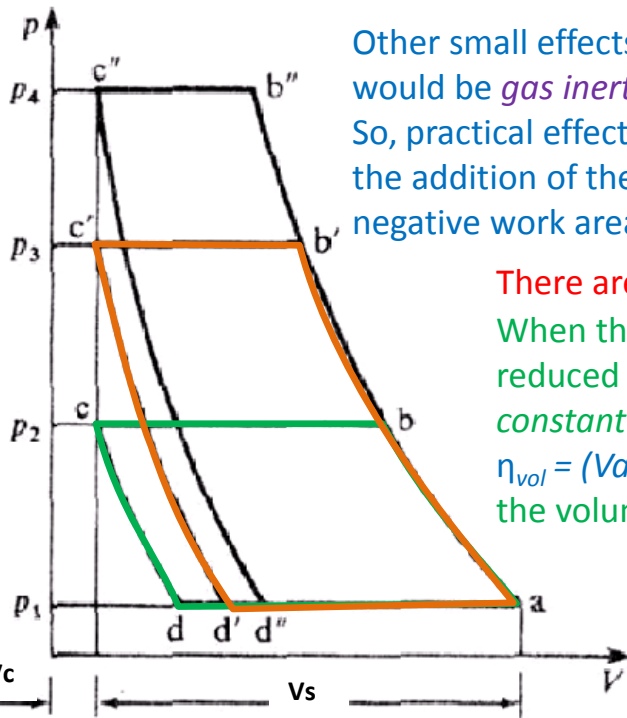
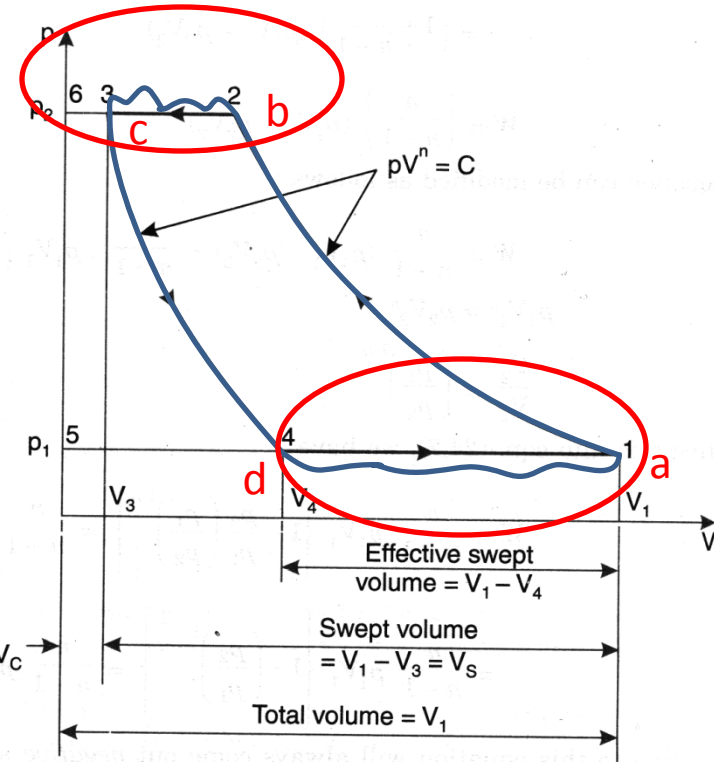
The greater is the clearance ratio through a reciprocating compressor, the greater will be the effect of the clearance volume since the clearance air will expand through greater volume before intake condition is reached.

Reciprocating Compressor

Actual p-V (indicator diagram for single stage compressor

At point 4, the clearance air actually reduced to atmospheric pressure. The inlet valve in practice will not open. Reason : (i) inlet valve inertia (ii) there must be a pressure difference across the inlet valve in order to move it. Thus pressure drop away until the valve is forced off its seat. Some *valve bounce* will then set in (wavy line) Therefore intake will become near enough steady at some pressure *below atmospheric pressure*. The negative pressure difference, i.e. *intake depression* settles naturally.

Similar situation occurs at point 2. There is a constant pressure rise, followed by *valve bounce* and the pressure then settles at some pressure *above external delivery pressure* (i.e. Receiver tank pressure).



Other small effects at inlet and delivery would be *gas inertia* and *turbulence*. So, practical effects are responsible for the addition of the two small ripple negative work areas shown in figure.

There are certain disadvantages to increase the delivery pressure to a high value. When the delivery pressure is increased to p_3 , the volume of the *fresh air induced* is reduced from $(Va - Vd)$ to $(Va - Vd')$, and so on, whereas swept volume V_s is remains constant. Since the volumetric efficiency is given by $\eta_{vol} = (Va - Vd) / V_s$ the volumetric efficiency decreases with increasing delivery pressure.

This situation can be improved by performing multistage compression process.

Reciprocating Compressor

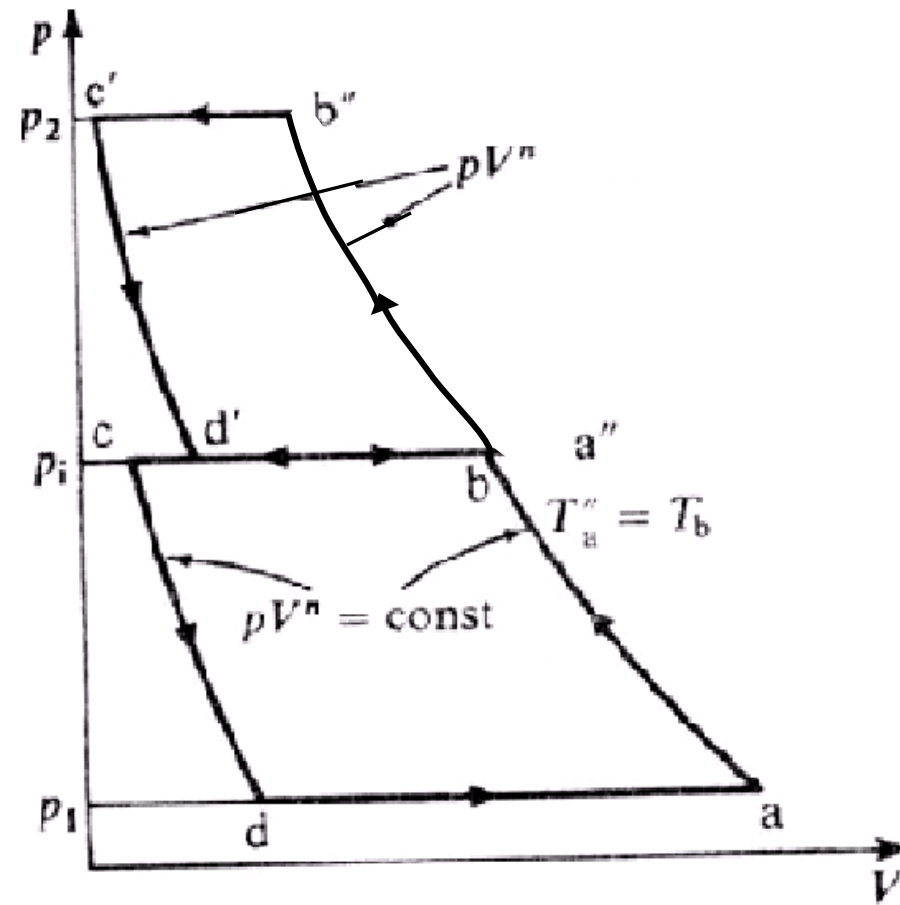
Multistage compression

Observation

After the first stage compression, the air is passed into a smaller cylinder, in which it is further compressed to desired final pressure. The cycle assumes that the delivery process of the first stage and the induction process of the second stage take place at the same pressure p_i .

Advantage

- Each cylinder works with lower pressure ratio. Thus the operational safety of the compressor is improved.
- **The overall volumetric efficiency, η_v increases.**
- **Mass flow rate is increased**, as clearance air expansion is reduced and **effective swept volume of this cylinder is increased.**



- (i) Single stage Compressor, for delivery pressure upto 5 bar.
- (ii) Two stage Compressor, for delivery pressure between 5 to 35 bar
- (iii) Three stage Compressor, for delivery pressure between 35 to 85 bar.
- (iv) Four stage compressor, for delivery pressure more than 85 bar

Reciprocating Compressor

Multistage compression

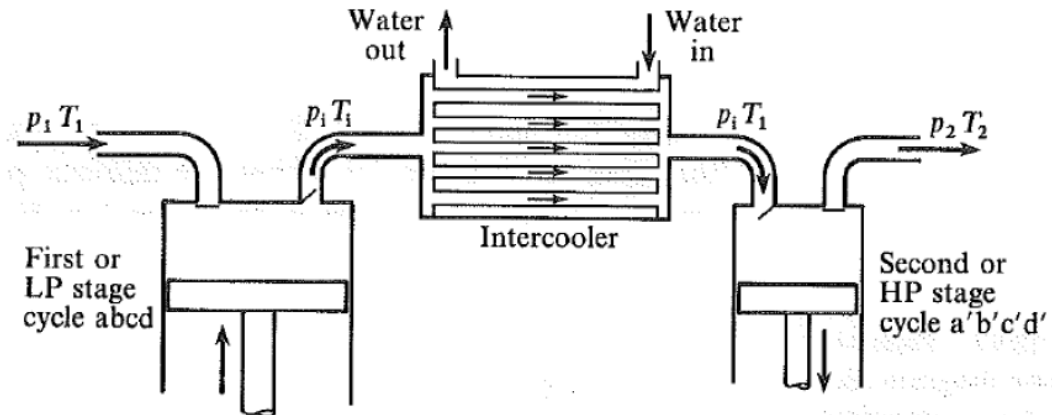
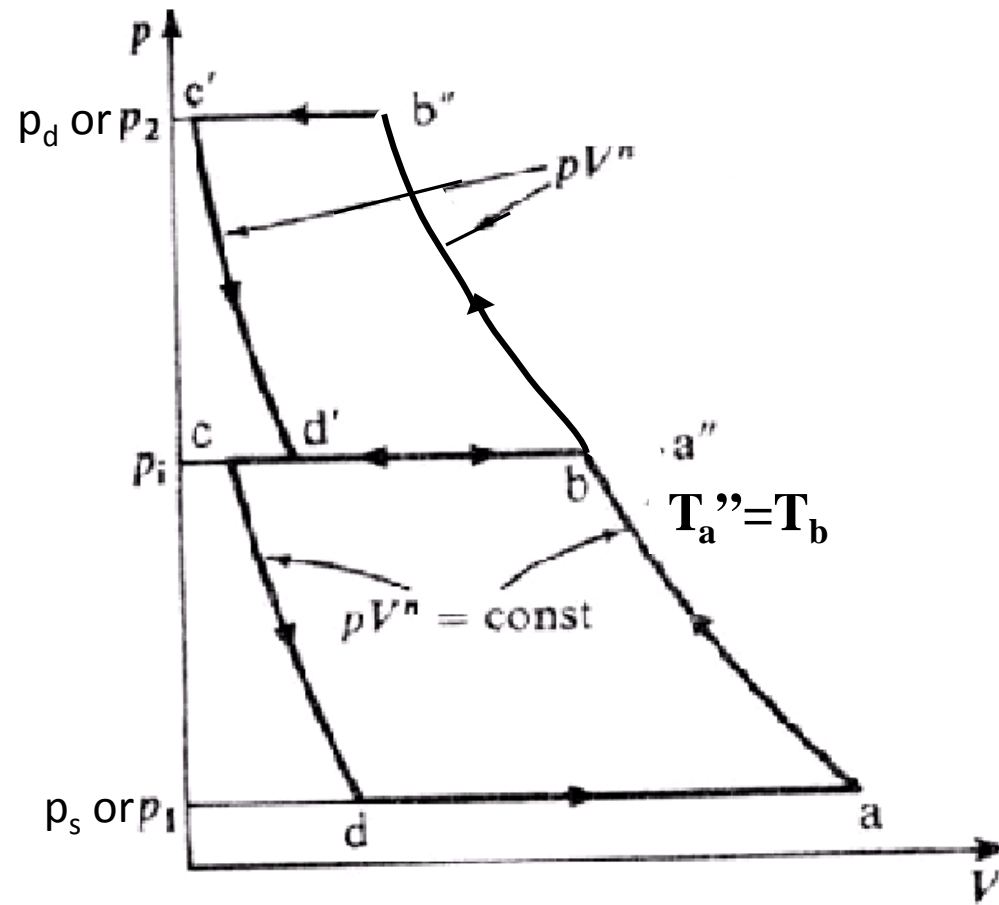
Indicated power for stage 1.

$$W = \left(\frac{n}{n-1} \right) mRT_a \left[\left(\frac{p_i}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Indicated power for stage 2.

$$W = \left(\frac{n}{n-1} \right) mRT_{a''} \left[\left(\frac{p_2}{p_i} \right)^{\frac{n-1}{n}} - 1 \right]$$

- With multistage compression, the air can be cooled as it is being transferred from one cylinder to the next, by passing it through an *intercooler*.
- The process of cooling the air is called the *intercooling* process.
- With intercooling process, temperature is reduced, therefore **internal energy of delivered air reduced**. Since energy must have come from the input energy required to drive the machine, this results in a **decrease in input work requirement for a given mass of delivered air**. Thus the power supplied to the compressor can be reduced.



Complexity of machine limits the number of stages.

Reciprocating Compressor

Multistage compression (Without Vc)

Single stage compressor: $W =$

$$\left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_5}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] = \left(\frac{n}{n-1}\right) m R T_1 \left[\left(\frac{p_5}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

Deliver temperature, $T_5 = T_1 \left(\frac{p_5}{p_1}\right)^{\frac{n-1}{n}}$

Two stage compressor, without intercooling: $W =$

$$\left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] + \left(\frac{n}{n-1}\right) p_4 V_4 \left[\left(\frac{p_5}{p_4}\right)^{\frac{n-1}{n}} - 1 \right]$$

Deliver temperature same as above.....

Two stage compressor,

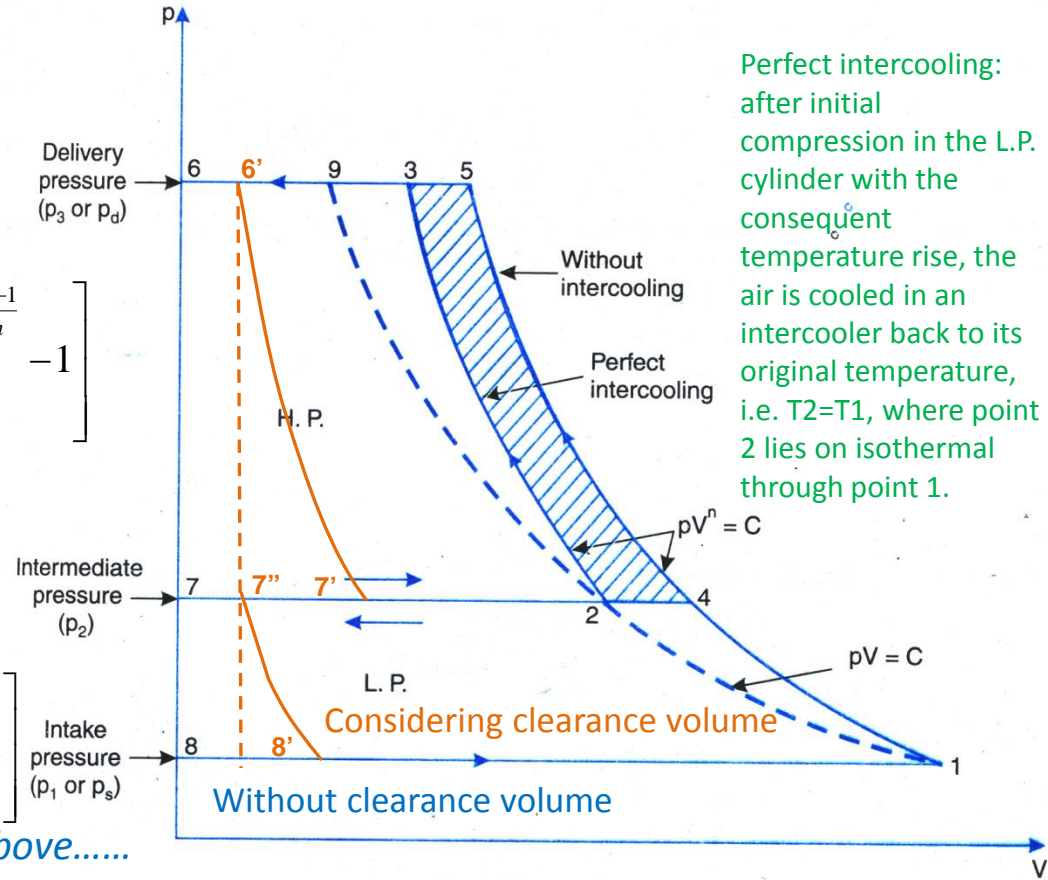
With perfect intercooling: $W =$

$$\left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] + \left(\frac{n}{n-1}\right) p_2 V_2 \left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1 \right]$$

Deliver temperature, $T_3 = T_2 \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} = T_1 \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}$, where $T_2 = T_1$

if $T_2 = T_1, p_2 V_2 = p_1 V_1, p_4 = p_2$

$$W = \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2 \right]$$



Perfect intercooling: after initial compression in the L.P. cylinder with the consequent temperature rise, the air is cooled in an intercooler back to its original temperature, i.e. $T_2 = T_1$, where point 2 lies on isothermal through point 1.

2453 = Work savings occurs

Reciprocating Compressor

Multistage compression (Without Vc)

Condition for minimum work done:

It is observed that an intermediate pressure $p_2 \rightarrow p_1$, then area 2453 $\rightarrow 0$. Also $p_2 \rightarrow p_3$, then area 2453 $\rightarrow 0$. This means, therefore there exists an intermediate pressure p_2 which makes area 2453 maximum and W minimum.

For minimum W : $\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}$ must be minimum.....

$$\frac{d \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} \right]}{dp_2} = 0 \quad \text{i.e.} \quad \frac{p_2}{p_1} = \frac{p_3}{p_2}$$

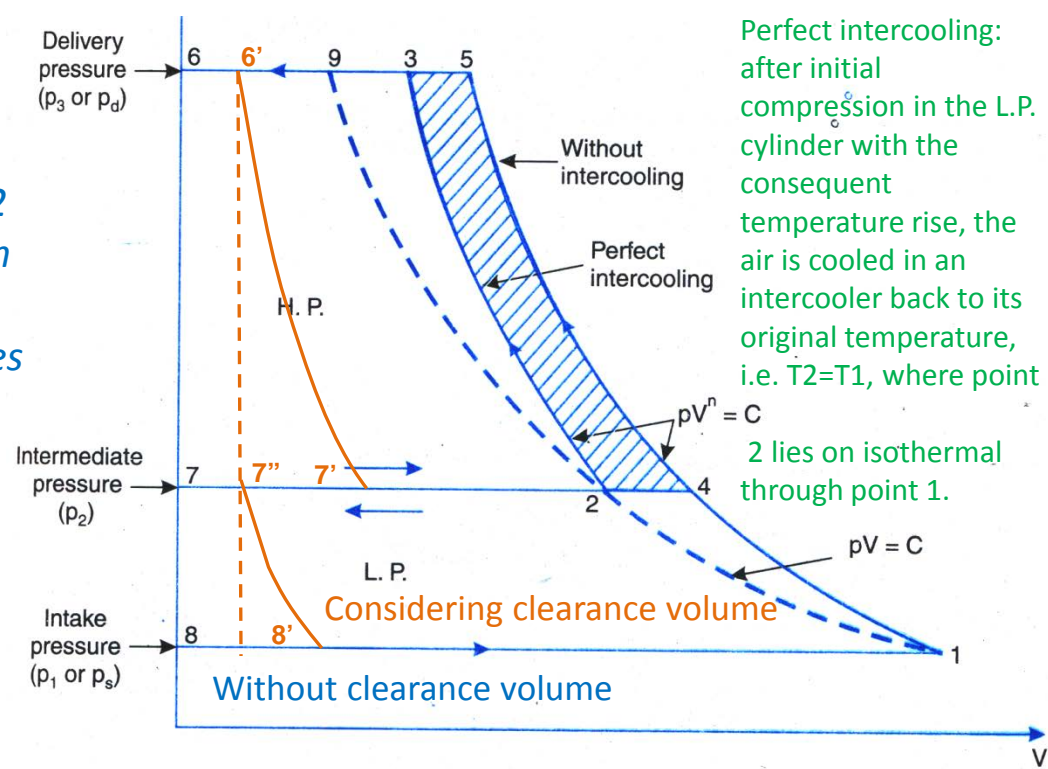
$$W = \left(\frac{2n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] = \left(\frac{2n}{n-1}\right) p_1 V_1 \left[\left(\frac{\sqrt{p_3 p_1}}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \left(\frac{2n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right] \text{ or } \left(\frac{xn}{n-1}\right) p_1 V_1 \left[\left(\frac{p_{(x+1)}}{p_1}\right)^{\frac{n-1}{xn}} - 1 \right] \text{ for } x \text{ stage}$$

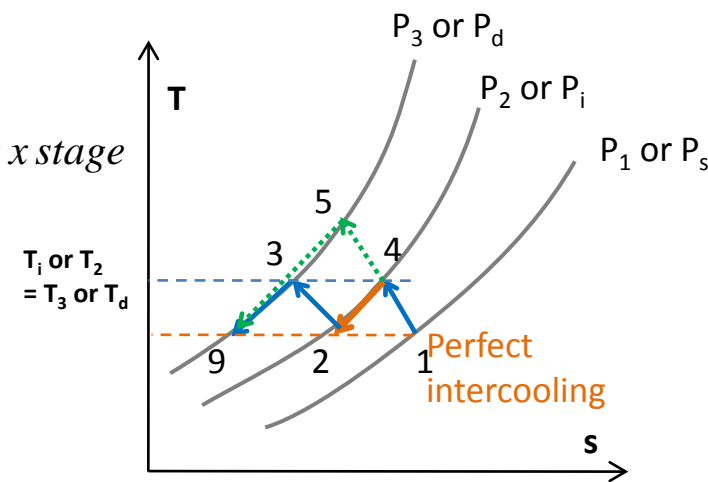
Where $p_{(x+1)}/p_1$ is the pressure ratio through compressor at x stage.

$$\text{Isothermal power } P_{\text{iso}} = mRT_1 \ln \left(\frac{p_3}{p_1} \right),$$

$$\text{Heat Transferred in intercooler} = mc_p (T_4 - T_2) = mc_p (T_4 - T_1)$$



Perfect intercooling: after initial compression in the L.P. cylinder with the consequent temperature rise, the air is cooled in an intercooler back to its original temperature, i.e. $T_2 = T_1$, where point 2 lies on isothermal through point 1.



Rotary Compressor:

PD type:

- (i) Lobe type (Roots blower)*
- (ii) Vane*
- (iii) Screw*
- (iv) Etc.*

Continuous Flow Compressor:

- (i) Centrifugal***
- (ii) axial*

Centrifugal Compressor

- Centrifugal compressors accelerates the velocity of the gases (increases kinetic energy) which is then converted into pressure as the air flow leaves the volute and enters the discharge pipe.
- Deliver much higher flow rates than positive displacement compressors
- For low pressure ratios ($< 4:1$), if higher pressure ratio with larger unit – prefer axial flow compressor
- Usually operate at speeds $> 3,000$ rpm.
- Smaller length, contaminated atmosphere doesn't affect the performance
- Disadvantages- larger frontal area and lower maximum efficiency

Basic Components

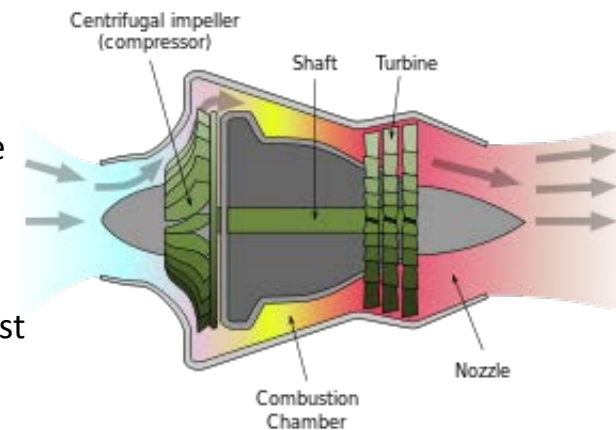
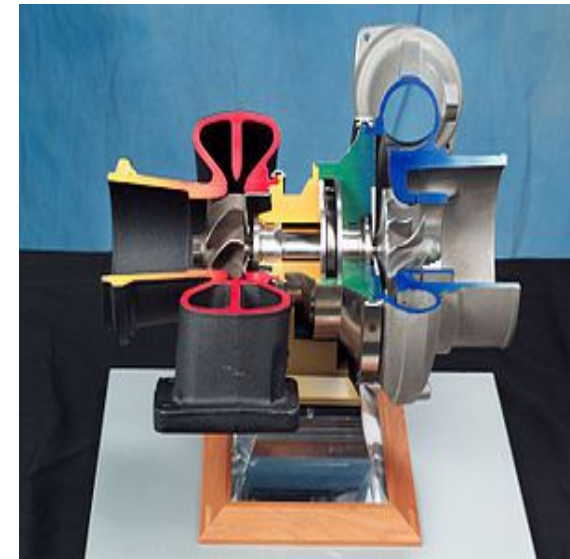
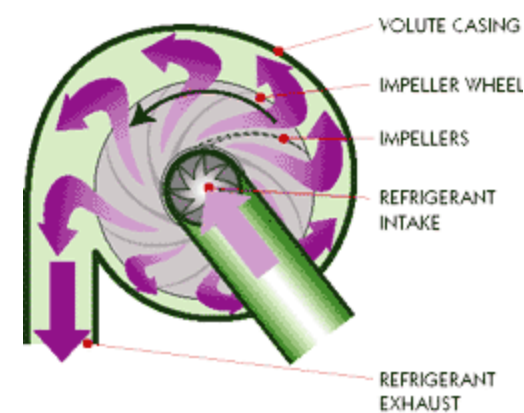
- Impellers, Vanes, Volutes, Suction Eyes, Discharge lines, Diffuser Plates, Seals, Shaft, Casing
- Suction Vane Tips = Part of the impeller vane that comes into contact with air first.
- Discharge Vane Tips = Part of the impeller vane that comes into contact with air last

Applications

Most well-known centrifugal compressor applications are gas turbines and turbochargers.

Either or both axial and centrifugal compressors are used to provide compressed air to Modern gas turbines which operate on the Brayton cycle. The types of gas turbines that most often include centrifugal compressors include turboshaft, turboprop, auxiliary power units, and micro-turbines.

Centrifugal compressors used in conjunction with reciprocating internal combustion engines are known as turbochargers if driven by the engine's exhaust gas and turbo-superchargers if mechanically driven by the engine.



Centrifugal Compressor

Inlet

The inlet to a centrifugal compressor is typically a simple pipe. It may include features such as a valve, stationary vanes/airfoils (used to help swirl the flow) and both pressure and temperature instrumentation.

Centrifugal impeller

The key component that makes a compressor centrifugal is the centrifugal impeller. It is the impeller's rotating set of vanes (or blades) that gradually raises the energy of the working gas. This is identical to an axial compressor with the exception that the gases can reach higher velocities and energy levels through the impeller's increasing radius. In many modern high-efficiency centrifugal compressors the gas exiting the impeller is traveling near the speed of sound.

Impellers are designed in many configurations including "open" (visible blades), "covered or shrouded", "with splitters". Most modern high efficiency impellers use "back sweep" in the blade shape. Euler's pump and turbine equation plays an important role in understanding impeller performance.

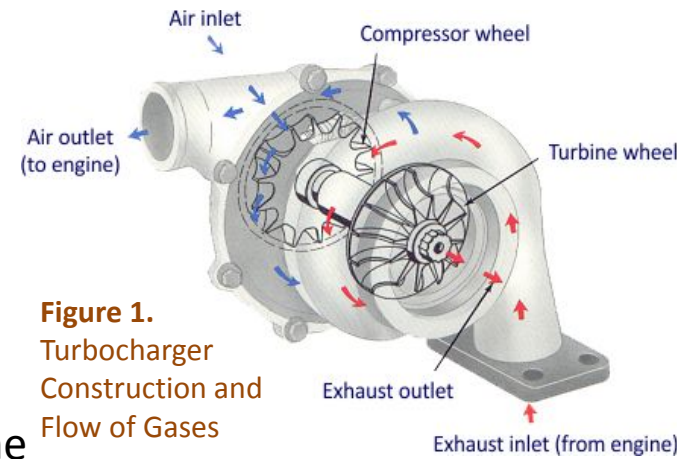


Figure 1. Turbocharger Construction and Flow of Gases



Figure 2. open impeller



Figure 3. full length splitter impeller

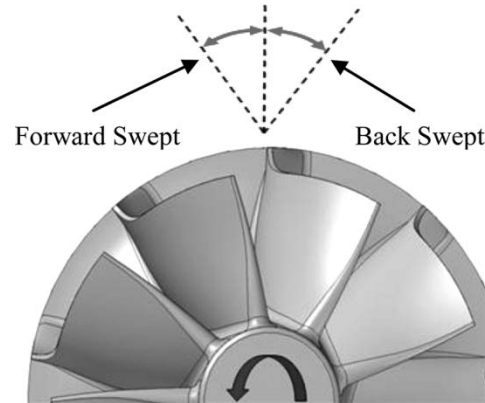


Figure 5. backsweep impeller



Figure 4. shrouded impeller

Centrifugal Compressor

Diffuser

The next key component to the simple centrifugal compressor is the diffuser. Downstream of the impeller in the flow path, it is the diffuser's responsibility to convert the kinetic energy (high velocity) of the gas into pressure by gradually slowing (diffusing) the gas velocity. Diffusers can be vane less, vaned or an alternating combination.

Hybrid versions of vaned diffusers include: wedge, channel, and pipe diffusers. There are turbocharger applications that benefit by incorporating no diffuser. Bernoulli's fluid dynamic principle plays an important role in understanding diffuser performance.

Collector / Casing

The collector of a centrifugal compressor can take many shapes and forms. When the diffuser discharges into a large empty chamber, the collector may be termed a *Plenum*. When the diffuser discharges into a device that looks somewhat like a snail shell, bull's horn or a French horn, the collector is likely to be termed a *volute* or *scroll*. As the name implies, a collector's purpose is to gather the flow from the diffuser discharge annulus and deliver this flow to a downstream pipe. Either the collector or the pipe may also contain valves and instrumentation to control the compressor.

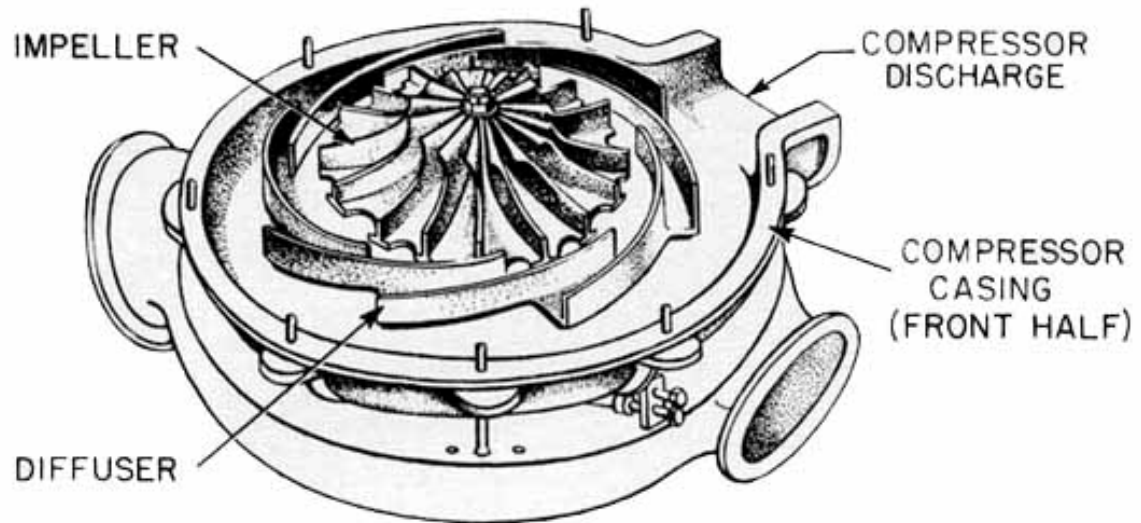


Fig. 7. Centrifugal compressor in turbosupercharger

Centrifugal Compressor (steady flow)

- Velocity encountered in the centrifugal compressor are very large, therefore total head quantities should be considered while analyzing centrifugal compressor.
- Consider, a horizontal passage of varying area through which air is flowing. Applying **steady flow equation** to the system we get

$$m_1 \left(u_1 + p_1 v_1 + \frac{V_1^2}{2} + g z_1 \right) + Q = m_2 \left(u_2 + p_2 v_2 + \frac{V_2^2}{2} + g z_2 \right) + W$$

for 1 kg of air flow (assuming **no external heat transfer and work done**) the expression becomes...

$$u_1 + p_1 v_1 + \frac{V_1^2}{2} = u_2 + p_2 v_2 + \frac{V_2^2}{2}$$

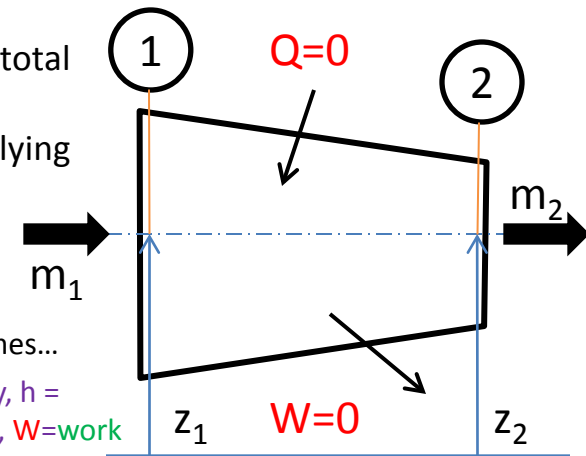
u = internal energy, v = volume, p = pressure, V = velocity, h = enthalpy, c_p = specific heat at constant pressure, Q = heat, W = work

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}$$

$$c_p T + \frac{V^2}{2} = \text{const}$$

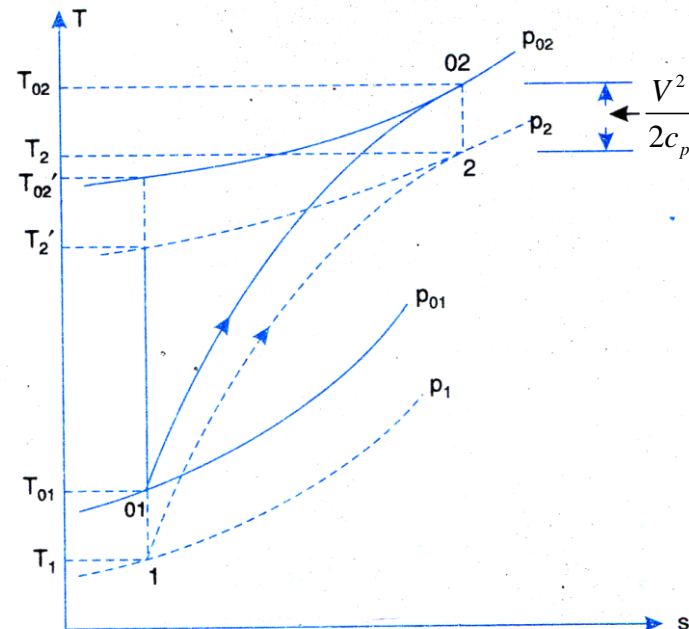
$P \cdot v$ represents the displacement or flow energy.
 $V^2/2$ represents the kinetic energy.
 $g \cdot Z$ represents the potential energy.



- T is static temperature, measured by thermometer when the thermometer is moving at the air velocity. **If moving air is brought to rest under reversible condition**, total KE of air is converted into 'u', further heat energy, increasing the temperature and pressure of the air. This temperature and pressure of the air is known as "stagnation" or "total head" temperature and pressure and it is denoted by 'o'.

$$c_p T + \frac{V^2}{2} = c_p T_o \quad T_o - T = \frac{V^2}{2c_p} \quad h_o - h = \frac{V^2}{2} \quad \frac{p_o}{p} = \left(\frac{T_o}{T} \right)^{\frac{\gamma}{\gamma-1}}$$

- Ideal reversible adiabatic process** is called **isentropic process** ($s = \text{const}$). But, **during the adiabatic compression** in a rotary compressor, there is friction between molecules of air and between air and blade passages, eddies formation and shocks at entry and exit. **These factors cause internal generation of heat and consequently the maximum temperature reached would be more than that for adiabatic compression ($T_{01} \rightarrow T_{02}$)**. This result in a progressive increase in entropy. Such a process through adiabatic (**no heat transfer**) is not isentropic. Again, **the heat generated by friction etc., may be removed continuously with the result that the process might not involve any entropy change ($T_{01} \rightarrow T_{02'}$)**. The process would be isentropic but not adiabatic as heat has been transferred.



Centrifugal Compressor (steady flow)

Isentropic efficiency – ratio of isentropic temperature rise to actual temperature rise or ratio of isentropic to actual compression work.

$$\eta_{isen} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} = \frac{T'_2 - T_1}{T_2 - T_1} = \frac{c_p (T'_{02} - T_{01})}{c_p (T_{02} - T_{01})} = \frac{\text{Isentropic Work}}{\text{Actual Work}}$$

During compression, work has to be imparted to the impeller (i.e. -ve).

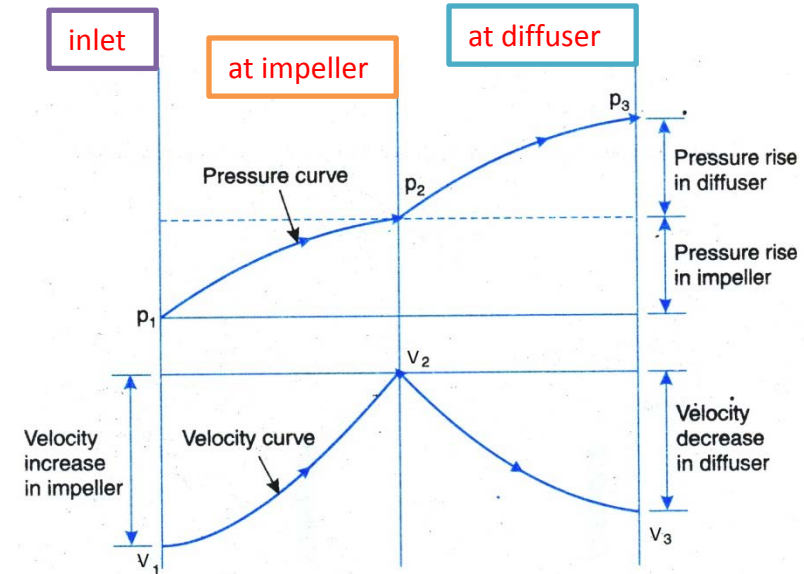
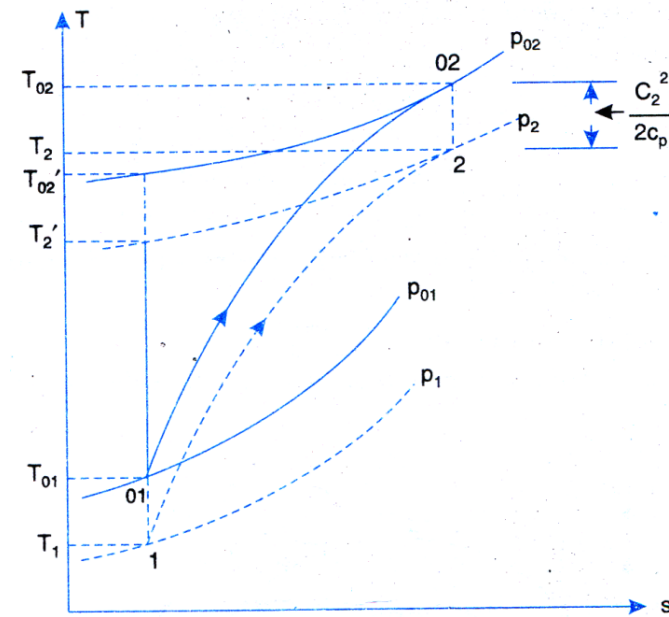
Then the energy balance equation around the impeller will be

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2} - W \quad c_p T_{01} + \frac{V_{01}^2}{2} = c_p T_{02} + \frac{V_{02}^2}{2} - W$$

$W = c_p (T_{02} - T_{01})$ Thus work input is the product of specific heat at constant pressure and temperature rise (for both adiabatic and isentropic process).

Status of P & V during Working Process

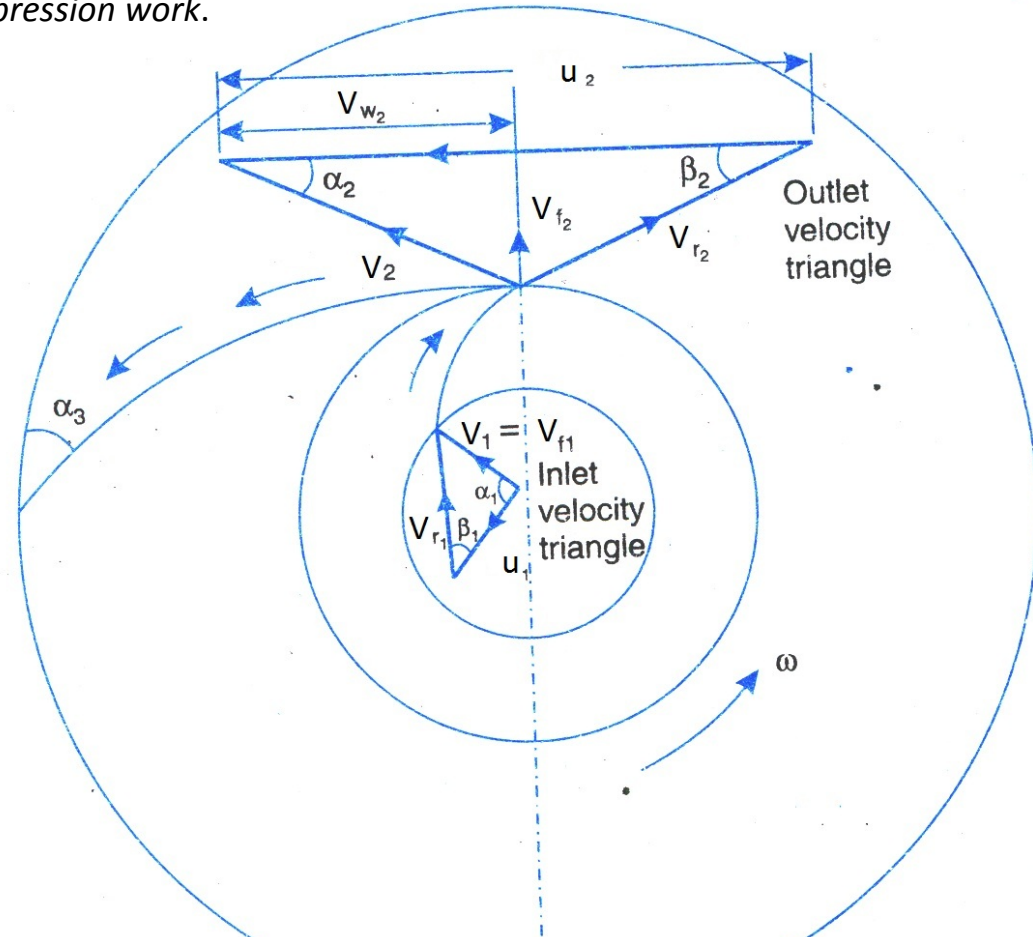
Air enters at the eye of the impeller at a mean radius r_m with low velocity V_1 and atmospheric pressure p_1 . Depending upon the centrifugal action of the impeller, the air moves radial outwards and during its movement it is guided by the impeller vanes. The impellers transfers the energy of the drive to the air causing a rise both static pressure p_2 and temperature T_2 , and increase in velocity V_2 . The work input equals the rise in total temperature. Air now enters the diverging passage 'diffuser' where it is efficiently slow down V_3 . The KE is converted into pressure energy with the result that there is a further rise in static pressure p_3 . In practice nearly half of the total pressure is achieved in impeller and remaining part in diffuser. A pressure ratio 4 : 1 can be achieved with single stage centrifugal compressor, for higher ratio multistage compressor is used.



Centrifugal Compressor (Velocity Diagram)

Isentropic efficiency – ratio of isentropic temperature rise to actual temperature rise or ratio of isentropic to actual compression work.

- V_1 = absolute velocity of fluid at inlet
- u_1 = Mean blade velocity at inlet
- v_{r1} = relative velocity of fluid at inlet
- v_{w1} = velocity of whirl (tangential) at inlet
- v_{f1} = velocity of flow at inlet
- u_2 = Mean blade velocity at outlet
- v_{r2} = relative velocity of fluid at outlet
- V_2 = absolute velocity of fluid at outlet
- v_{w2} = velocity of whirl at outlet
- v_{f2} = velocity of flow at outlet
- α_1 = exit angle from the guide vane or inlet angle of the guide vane
- β_1 = inlet angle to the rotor or impeller or angle between v_{r1} with the direction of motion of vane at inlet
- β_2 = outlet angle to the rotor or impeller or angle between v_{r2} with the direction of motion of vane at inlet
- α_2 = inlet angle to diffuser



It assumes that entry of the air is 'axial' therefore the whirl V_{w1} is zero, and $V_1 = V_{f1}$. To avoid shock at entry and exit the blade must be parallel to the relative velocity of air at inlet or outlet and β_1 and β_2 are the impeller blade angle at inlet and outlet. The diffuser blade angle must be parallel to the absolute velocity of air from the impeller (V_2), therefore α_2 is the diffuser blade angle at inlet and α_3 will be the diffuser blade angle at outlet. If the discharge from the diffuser is circumferential, then angle at outlet (α_3) should be as small as possible.

Centrifugal Compressor (Equations)

Work done by the impeller (Euler's Work)

$$W = V_{w2}u_2 - V_{w1}u_1 = h_{02} - h_{01} = c_p (T_{02} - T_{01}) \quad \text{If working fluid enters radially, } V_{w1}=0$$

Using inlet and outlet velocity triangle

$$W = \frac{\overset{\text{term-I}}{V_2^2 - V_1^2}}{2} + \frac{\overset{\text{term-II}}{V_{r1}^2 - V_{r2}^2}}{2} + \frac{\overset{\text{term-III}}{u_2^2 - u_1^2}}{2}$$

Power required per impeller for \dot{m} kg of air flow in one second:

$$P = \frac{\dot{m} V_{w2} u_2}{1000} \text{ kW}$$

If the blade is radial (ideal case), $V_{w2}=u_2$, and $W=V_2^2$ Since the air cannot leave the impeller at a velocity greater than the impeller tip velocity, the maximum work supplied per kg of air per second is given by the above equation.

Term I = **increase in KE** of 1 kg of working fluid in impeller, that has converted into pressure energy in the diffuser

Term II = **pressure rise** in the impeller due to '**diffusion action**', as relative velocity decreases from inlet to outlet

Term III = **pressure rise** in the impeller due to '**centrifugal action**', as working fluid enters at a lower diameter and comes out at a higher diameter

Considering the steady flow at the inlet and outlet of the impeller, assuming the **heat transfer during the flow of air through the impeller is zero i.e. adiabatic.**

$$h_1 + \frac{V_1^2}{2} + W = h_2 + \frac{V_2^2}{2} \quad W = \left(h_2 + \frac{V_2^2}{2} \right) - \left(h_1 + \frac{V_1^2}{2} \right) = c_p \left(T_2 + \frac{V_2^2}{2c_p} \right) - c_p \left(T_1 + \frac{V_1^2}{2c_p} \right) = c_p (T_{02} - T_{01})$$

$$c_p T + \frac{V^2}{2} = \text{const} = c_p T_0 + \frac{V_0^2}{2}$$

$$W = c_p T_{01} \left(\frac{T_{02}}{T_{01}} - 1 \right) = c_p T_{01} \left(T_2 \left(\frac{P_{02}}{P_2} \right)^{\frac{\gamma-1}{\gamma}} / T_1 \left(\frac{P_{01}}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) = c_p T_{01} \left(\left(\frac{P_{02}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) = c_p T_{01} \left(r_{p0}^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

r_{p0} = pressure ratio based on stagnation pr.

$$W = V_2^2 = c_p T_{01} \left(r_{p0}^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

In most of the practical problem $V_1=V_2$, therefore

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$W = c_p T_1 \left(r_p^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad \frac{P_2}{P_1} = \left(\frac{V_2^2}{2c_p T_1} + 1 \right)^{\frac{\gamma-1}{\gamma}}$$

r_p = pressure ratio based on static pr.

Power input to the compressor depends upon:

- (i) Mass flow of air through the compressor
- (ii) Total temperature at the inlet of the compressor
- (iii) Total pressure ratio of the compressor which depends upon the square of the impeller tip velocity.

Centrifugal Compressor (Equations)

Width of the blades of impeller and diffuser

$$\dot{m} = \frac{\text{Volume of the air flowing per second}}{\text{Volume of 1 kg of air}} = \frac{2\pi r_1 b_1 V_{f1}}{v_1} = \frac{2\pi r_1 b_1 V_1}{v_1}$$

$$\dot{m} = \frac{2\pi r_2 b_2 V_{f2}}{v_2} \quad \text{Similarly at outlet}$$

$$\dot{m} = \frac{2\pi r_d b_d V_{fd}}{v_d} \quad \text{Similarly at diffuser, 'd' for diffuser}$$

Isentropic efficiency

$$\eta_{isen} = \frac{\text{Isentropic Work}}{\text{Actual Work}} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} = \frac{c_p (T'_{02} - T_{01})}{c_p (T_{02} - T_{01})} = \frac{c_p (T'_2 - T_1)}{c_p (T_2 - T_1)}$$

$$\eta_{isen} = \frac{T'_2 - T_1}{T_2 - T_1} = \frac{T'_2/T_1 - 1}{T_2/T_1 - 1} = \frac{(p_2/p_1)^{(\gamma-1)/\gamma} - 1}{(p_2/p_1)^{(n-1)/n} - 1}, n > \gamma$$

Slip Factor (ϕ_s) – ratio of actual whirl component and the ideal whirl component (radial exit)

$$\phi_s = \frac{V_{w2}}{u_2} \quad \text{slip} = u_2 - V_{w2}$$

Pressure Coefficient (ϕ_p) – ratio of isentropic work to Euler work

$$\phi_p = \frac{c_p (T'_{02} - T_{01})}{u_2 V_{w2}} \quad \text{Work factor or power input factor } \phi_w = u_2 V_{w2}$$

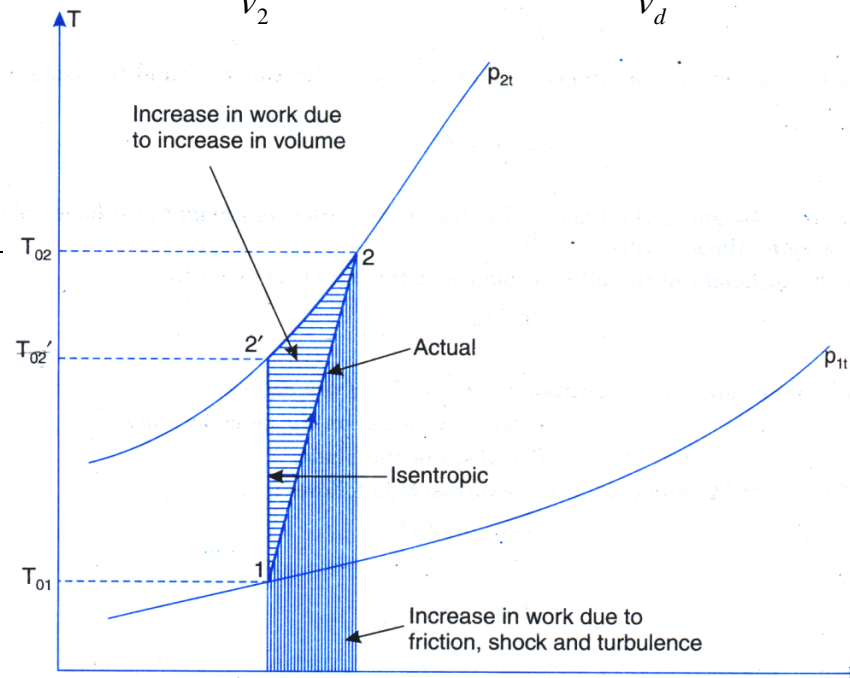
\dot{m} = mass of air flowing per second
 b_1 = width (or height) of impeller at inlet
 r_1 = radius of impeller at inlet
 $V_1 = V_{f1}$, as air trapped radially

If, n is number of blade in the impeller having 't' thickness

$$\dot{m} = \frac{(2\pi r_1 - nt)b_1 V_{f1}}{v_1}$$

$$\dot{m} = \frac{(2\pi r_2 - nt)b_2 V_{f2}}{v_2}$$

$$\dot{m} = \frac{(2\pi r_d - nt)b_d V_{fd}}{v_d}$$



At radial exit $u_2 = V_{w2}$

Centrifugal Compressor (Equations)

Diffuser efficiency

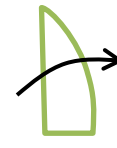
$$\eta_d = \frac{(p_2 - p_1)}{\frac{\omega}{2g}(V_1^2 - V_2^2)}$$

Suffix 1 and 2 represents upstream and downstream condition of diffuser, w =weight density

1. Backward blades – High head or pressure ratio
2. Radial vane – high outlet velocity, manufactured easily, free from complex bending stress, high pressure ratio



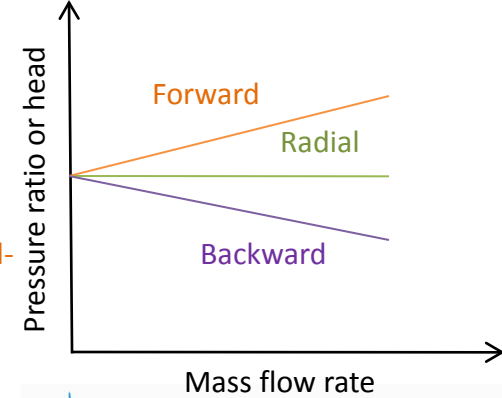
Backward-curved blades
 $\beta_2 < 90^\circ$



Radial-curved blades
 $\beta_2 = 90^\circ$

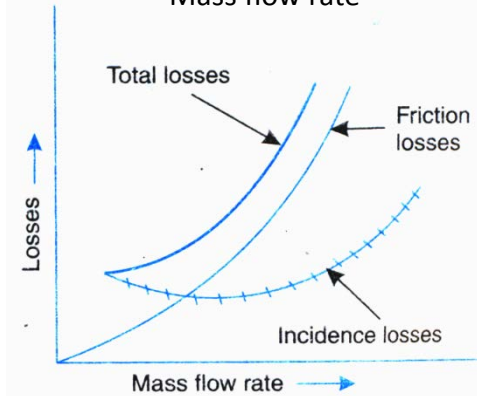


Forward-curved blades
 $\beta_2 > 90^\circ$



Losses in Centrifugal Compressor

1. **Friction loss** frictional losses are due to both skin friction and boundary layer separation. It is proportional to both V^2 and m^2
2. **Incidence loss** – During the off-design conditions, the direction of relative velocity of fluid at inlet does not match with the inlet blade angle and therefore fluid cannot enter the blade passage smoothly by gliding along the blade surface. It is proportional to $C_D V^2$, where C_D is drag coefficient.



Selection of Compressor geometries

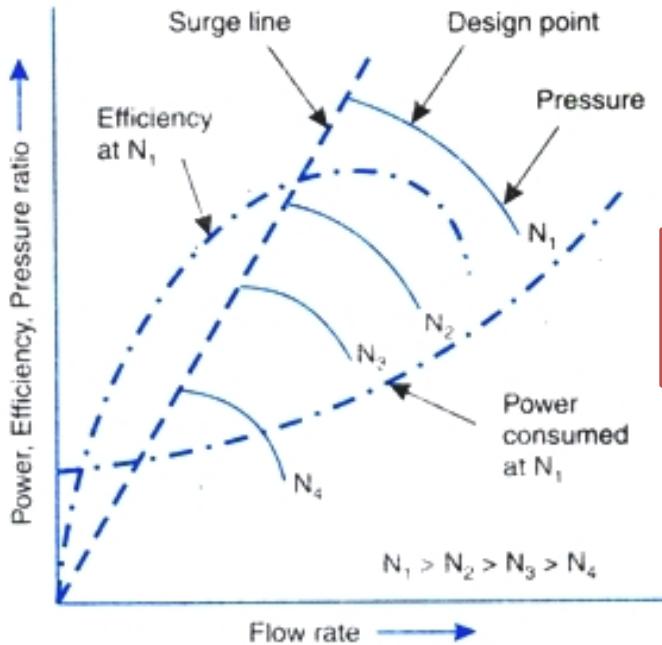
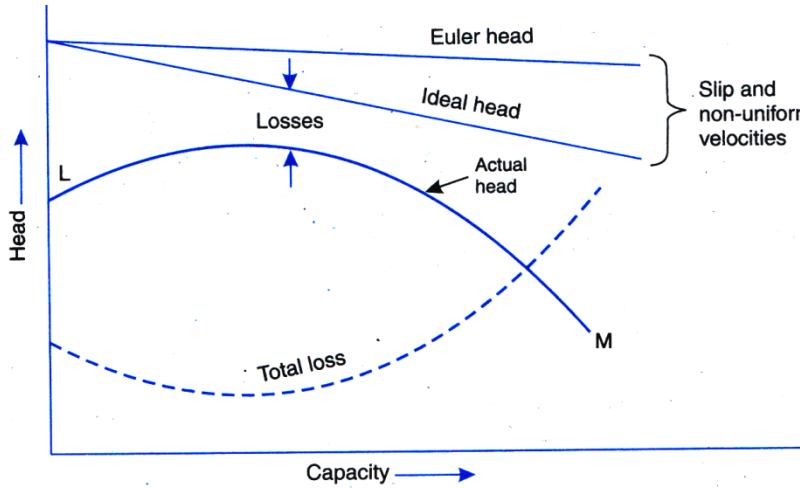
1. **Number of Blade in impeller:** Can be chosen by experience corresponding to requirements. Usually experiments shows that it varies from 18 to 22 for radial bladed diameter from 25 to 36 cm
2. **Blade Angle-** outlet angle influence the inlet angle, usually inlet angle varies from 30 to 35 degree.
3. **Impeller diameter** – $u_2 = \pi D_2 N / 60$ after knowing the tip diameter, the inlet diameter is calculated from the tip diameter ratio. Generally D_2/D_1 varies from 1.6 to 2.0
4. **Impeller width** - If b_1 and b_2 are the blade width at inlet and outlet of the impeller, then neglecting the thickness of blades it is calculated by the equation.
 $m = \pi d_1 b_1 V_{f1} \rho_1 = \pi d_2 b_2 V_{f2} \rho_2$ Generally $V_{f1} = V_{f2}$
5. **Impeller Material** – forged or die casted of low silicon aluminum alloy.
6. **Vaneless diffuser** – The function of vaneless diffuser or space is to stabilize the flow for shockless entry into a bladed diffuser and to invert some portions of K.E. into pressure energy. The diameter ratio of vaneless to impeller tip diameter (D_3/D_2) varies from 1/0.06 to 1.12

Since the flow in the vaneless diffuser is assumed to be logarithmic spiral, hence $\alpha_2 = \alpha_3$. Generally $b_2 = b_3$ = width of vaneless diffuser. In some cases $b_3 > b_2$

Centrifugal Compressor

Compressor Characteristics curve

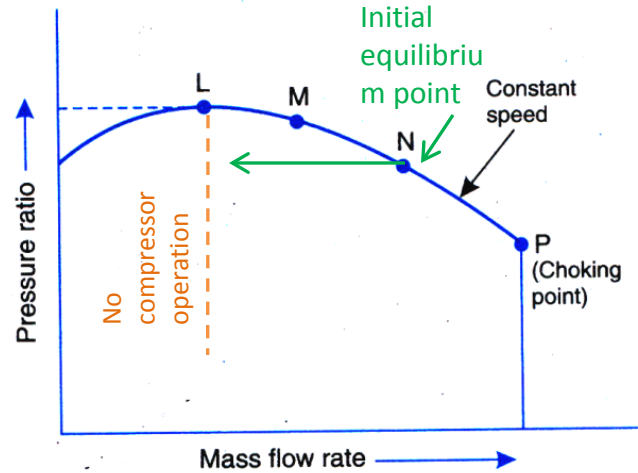
Inlet loss, friction, separation loss, losses in diffuser



Compressor Performance curve

Surging and Choking

Suppose compressor is running in equilibrium condition at N, If flow is restricted (i.e. low mass flow rate) by valve or by any means resistance of flow is increased, the equilibrium point moves to M. If flow still reduces, operating point moves left to L (max pr. ratio). If flow further reduced, pressure ratio will reduce. At this moment, there is a higher pressure downstream system than compressor delivery. So flow stops or may even reverse.



After a short duration, compressor starts to deliver fluid (say N). Pressure starts to increase and operating point moves towards left from right again, and after point 'L' it cuts and the cycle will be repeated with high frequency. It is called **surging**.

At a constant rotor speed, the tangential velocity component (V_{w2}) at the impeller tip remains constant. With the increase in mass flow the pressure ratio decreases and hence the density is decreased. Consequently, the radial velocity (V_{r2}) is increased considerably, which increases the absolute velocity and incidence angle at diffuser vane tip. Thus there is a rapid progression towards **choking state**. Beyond this compressor cannot be operated.

Axial Flow Compressor

Composed of a rotor that has rows of fanlike blades.

- In industry, axial compressors are used a lot high flows and pressures are needed.
- Air flow is moves along the shaft.
- Rotating blades attached to a shaft push air over stationary blades called stators.
- Stator blades are attached to the casing.
- As the air velocity is increased by the rotating blades, the stator blades slow it down. As the air slows, kinetic energy is converted into pressure.
- air velocity increases as it moves from stage to stage until it reaches the discharge.
- Multi-Stage axial compressors can generate very high flow rates and discharge pressures.
- Axial compressors are usually limited to 16 stages (due to temperature/material limitations)
- Pound for pound, axial compressors are lighter, more efficient, and smaller than centrifugal compressors.